

# Patterns under Quantum Confined Stark Effect

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We have studied pattern formation under QCSE and found different patterns with complex structure of the electron and hole wave functions which gives rise to nonuniform dipolar patterns of the electric charge inside the QW layer. The results obtained indicate spontaneous breaking of the transversal invariance.

The Quantum Confined Stark Effect<sup>1</sup> (QCSE) arises when a strong electric field is applied to a quantum well (QW) heterostructure. This field affects the energies and wave functions of electron and hole subbands, and exciton states. QCSE is highly sensitive to the photo-generation of electrons and holes. Electrons and holes screen the applied field and produce changes in the optical spectra near the fundamental edge of absorption. Furthermore, when the spectrum of the illuminating light is tuned into the region between exciton and interband absorption, light absorption becomes bistable.

This kind of optical bistability is observed for different quantum structures with two common characteristics: they all show bistable behavior despite the different character of the relaxation and the transport of electrons and holes, and all of them are layered structures.<sup>2,3</sup> Within these structures, carrier motion and diffusion on the transversal directions couple the states at different points and eventually produce nonuniform patterns in the electron-hole plasma (regions with different absorption and electron-hole concentrations, and different configurations of the electrostatic field).<sup>3,4</sup>

Under conventional QCSE in QWs, the quantized vertical motion of carriers and their transversal motion are entirely uncoupled. The external field separates electrons and holes and produces a homogeneous charge dipole layer inside the well. In this case, the vertical and transversal degrees of freedom of the carriers are strongly coupled, leading to the appearance of a complex structure of wave functions and electric charges inside the layer.

Let us consider a single QW layer to which is applied an external electric in the growth direction (vertical) and a uniform photon source. The basic equations are: Schrödinger equation for electron and hole wave functions,

Poisson equation for the electrostatic potential and drift-diffusion equations for the two-dimensional electron and hole concentrations. The strong separation among the characteristic length scales of the problem for typical experimental conditions (the well width, the transversal electron wavelength, the screening length of the two-dimensional electron-hole gas and the characteristic length scale of the transversal patterns, determined in the analysis) allowed us to use the following approximations: (i) the total wave functions of the carriers are factorized to the product of the wave functions of the vertical and transversal motion; (ii) the transversal transport of carriers is quasi-classical; (iii) the wave functions of the quantized vertical motion, and the subband energies depend on the transversal coordinates parametrically; and (iv) the redistribution of the electron and hole concentrations is quasi-neutral,  $n \approx p$ .<sup>5</sup> In the first order approximation, the stationary system of equations is reduced to the following dimensionless equation:

$$-\nabla_{\mathbf{r}} \cdot \{ \mathcal{D}(n, q) \nabla_{\mathbf{r}} n \} = a(n, q) i - n \equiv R(n, q, \omega, i), \quad (1)$$

where  $\mathcal{D}$  is a function of the concentration  $n$  and the electric field  $q$ ,  $\mathbf{r}$  is the vector of transversal coordinates, and the right-hand side is the dimensionless representation of the generation and recombination rates, where  $a$  is the absorption factor and  $i$  is the dimensionless illuminating intensity.<sup>3,4,5</sup>

The remainder of the letter will discuss two type of solutions for Eq. 1: one-dimensional solutions and radial solutions (independent of the angular coordinate). The condition  $R(n, q, \omega, i) = 0$  gives transversally uniform solutions. It can be shown that there are intervals of intensities and electric fields with *three* branches of uniform solutions  $n = n(i, q)$ : low absorption branch (low concentration), high absorption branch (high concentration) and the middle branch. The bistable regime occurs in the interval  $i_l < i < i_h$ .<sup>5</sup>

Let us consider that the QW layer is infinite in the transversal direction. Then possible nonuniform solutions should tend to the stable fixed points. There are three types of such nonuniform solutions. To classify them we shall introduce a critical value of the intensity,  $i_c$ , which solves the equation  $\int_{n_l}^{n_h} dn' R(n', i_c) \mathcal{D}(n') = 0$ . The nonuniform solutions are homoclinic and heteroclinic orbits in the phase plane defined by Eq. (1): (i) *anti-soliton-like* patterns having a minimum (if  $i_l < i < i_c$ ); (ii) *kink-like* patterns (if  $i = i_c$ ); and (iii) *soliton-like* patterns having a maximum (if  $i_c < i < i_h$ ).

Now we consider the three basic patterns listed above. In Fig. 1-I we present density plots of the square of the electron wave functions ( $|\psi_e|^2$ ) inside the QW layer under these conditions. The breaking of the transversal translation symmetry can be seen clearly: the carrier wave functions become dependent on the transversal coordinate and are different for each pattern.

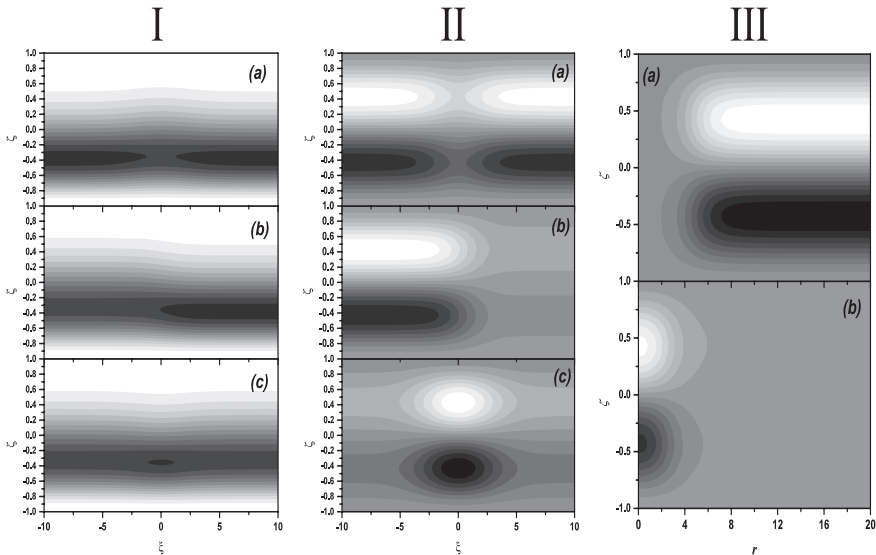


Figure 1: (I) Distribution of the square of the electrons wave function  $|\psi_e|^2$  for the three basic patterns described in the body of the text for an infinite transversal extension of the QW. Darker areas represent zones with higher density of states. (II) Electric charge distribution for the three basic patterns described in (I). Darker areas are zones with positive charge. (III) Electric charge distribution in the radial plane for anti-soliton case ( $i_l < i < i_c$ , a), and soliton case ( $i_c < i < i_h$ , b). Darker areas are zones with positive charge.

The wave function distributions are symmetric with respect to the center of the soliton and the anti-soliton patterns, whilst the kink-like pattern has an asymmetric wave function distribution. These distributions are due to different screening of the external field in different patterns. For example, the field of the soliton-like pattern is screened in the central region by the excess of electrons and holes. There the subband energies are higher and the wave function becomes flatter and uniformly distributed across the QW layer (Fig. 1-Ic). All these features correspond to local partial suppression of the QCSE. On the contrary, the QCSE is enhanced in the central part of the anti-soliton pattern (Fig. 1-Ia).

Fig. 1-II depicts the electric charge distribution  $\rho = n(|\psi_h|^2 - |\psi_e|^2)$  for the same basic patterns. The central horizontal line separates half of the QW layer with a negative charge from that with a positive charge. Notice that instead of a transversally uniform dipole layer we obtain a complex distribution of the electric charge, which adopts the form of a nonuniform dipole layer

inside the QW. In the case of the soliton-like pattern (Fig. 1-IIc), there is an excess of carriers in the central region of the pattern, where the dipole strength is maximal. For the anti-soliton pattern (Fig. 1-IIa), two symmetric regions depleted of electrons and holes appear in the central part of the QW. Fig. 1-IIb shows the charge configuration in the transition region between the states with high and low concentration of a kink-like pattern.

In the case of radial solutions, phase-portraits techniques cannot be used, but there are still two range of intensities with different behavior. As an example, Fig. 1-III presents the charge distribution in a radial plane. Similar situations to those of the one-dimensional solution appear.

In conclusion, we have studied pattern formation under QCSE and found different patterns with a complex structure of the electron and hole wave functions and of the electric charge inside the QW layer. The results obtained indicate the breaking of the transversal invariance, which can lead to a set of new optical effects and transport phenomena: changes in the selection rules for optical transitions, patterning of the transmitted light intensity, anisotropy of the conductivity of two-dimensional electron-hole plasma, etc.

## Acknowledgments

We are indebted to the Dirección General de Enseñanza Superior (Spanish Ministry of Education) for sabbatical support (VAK) and for financial support through grant PB94-0375. One of us (CAV) acknowledges the support of the Fundación General de la Universidad Carlos III de Madrid.

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