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# Quasiperiodic current and strange attractors in ac-driven superlattices

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Intriguing routes to chaos have been experimentally observed in semiconductor superlattices driven by an ac field. In this work, we analyze theoretically the time-dependent current in ac-driven superlattices. Experiments have shown that the current is quasiperiodic and its Poincaré maps are distorted and twisted for certain values of the ac intensity, at fixed dc voltage. This marks the appearance of very complex attractors and routes to chaos as the ac signal amplitude increases. We show that discrete well-to-well motions of domain walls during spiky high-frequency self-oscillations of the current play a crucial role in the origin of the experimentally observed distorted maps.

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Spatiotemporal chaos has been analyzed in fluid and chemical systems[1] as well as in solid-state systems.[2,3] Different routes to chaos, as the system is driven by a frequency incommensurate with its natural frequency, start from quasiperiodic attractors and end in chaos via frequency locking or directly. Interesting theoretical and experimental researches of this topic have been realized in semiconductor superlattices.[4–9]

Nonlinear dynamics of weakly coupled semiconductor superlattices (SLs) driven by dc and ac biases has been the research topic of many experimental and theoretical works.[10–17] The nonlinearity manifests itself in many physical situations as, for instance, in their transport properties under finite dc or ac bias. Under appropriate dc voltage, the current through the SL displays natural oscillations caused by creation, motion, and annihilation of domain walls (DW) in the SL.[14] The observed frequencies range from kHz to GHz. Superimposed on a smooth current oscillation, there appear faster current spikes whose frequencies are typically in the GHz regime.[14] Each spike reflects the well-to-well motion of DWs causing the self-oscillation, and therefore their frequency is related to the characteristic tunneling time. Numerical simulations show that DW may correspond to charge monopoles or dipoles,[15,16] although experimental evidence suggests that charge monopoles are responsible for self-oscillations observed in the available samples.[14] ac driven SLs display a rich dynamical behavior including quasiperiodic and chaotic oscillations with nontrivial phase space structure.[4,6–9]

Many studies fix the frequency of the ac drive as the golden mean number $(1+\sqrt{5})/2\approx 1.618$ times the frequency of the natural oscillations (i.e., the frequency ratio is an irrational number hard to approximate by rational numbers), which is convenient to obtain complex dynamical behavior. Then the system presents rich power spectrum, complex bifurcation diagrams, and different routes to chaos including quasiperiodicity, frequency locking, etc.[4,6,7,9] First return or Poincaré maps (PM) are used to analyze unambiguously the underlying attractors.[7] PMs usually consist of smooth loops for quasiperiodic attractors, and of a set of discrete points for frequency locking. More exotic PM resembling distorted double loops in the quasiperiodic case have been experimentally observed in the middle of the second plateau of a SL $I-V$ characteristic (at the onset of the second plateau, PM are smooth and not distorted).[6,7] These PM correspond to quasiperiodic nonchaotic strange attractors having a fractal phase space structure.[9,18,19] The origin of distorted maps was not understood at the time of their observation, although disorder and sample imperfections were invoked.[6] Luo *et al.*[7] showed that a combination of signals with different frequencies (whose origin was not ascertained) was needed to reproduce experimentally observed distorted double layer PM.

The aim of the present work is to establish theoretically that high-frequency current spikes play a relevant role in the occurrence of exotic PM. In turn, current spikes are due to the well-to-well motion of DW during each period of the self-oscillations. Thus distorted PM reflect DW motion in ac driven SLs.

Theoretical studies of nonlinear effects in weakly coupled SL are based upon discrete versions of Poisson and charge continuity equations plus appropriate boundary conditions. Early models were discrete drift models with a phenomenological fitting of sequential tunneling current between adjacent wells and boundary conditions (see Refs. 14, 15, and references therein; more complicated rate equation models with tunneling currents derived from simple quantum mechanical considerations were also used[15]). More recently, the sequential tunneling current has been obtained microscopically, and the electrostatics at the contacts has been microscopically modeled and included self-consistently.[12,13,16] Depending on SL configuration and contact doping, we know that undriven self-oscillations are due to recycling of either monopole DWs or dipole waves.[16] In the latter case, a dipole wave consists of two DWs, one corresponding to a charge accumulation layer and another one corresponding to a depletion layer. Between these DWs there is a high-field region encompassing several wells, and the dipole wave travels through the whole SL. Recycling of dipole waves causes current self-oscillations similar to the well-known Gunn effect in bulk semiconductors.

In this paper, we have analyzed the tunneling current through a dc+ac biased SL by means of the microscopic model described in detail in Refs. 12,13. We have considered a 50-period SL consisting of 13.3 nm GaAs wells and 2.7 nm

 

AlAs barriers, as described in Ref. 14. Doping in the wells and in the contacts are $N_w = 2\times10^{10}$ cm$^{-2}$ and $N_c = 2\times10^{18}$ cm$^{-3}$, respectively. With these doping values, self-oscillations are due to recycling of monopole DWs. We choose not to analyze the original 9/4 sample where the experiments were performed.[6] In this sample, the second plateau of the $I-V$ characteristics ends at a resonant peak due to $\Gamma-X$ tunneling. In the simpler SL configuration chosen here, the $X$ plays no role at the first and second plateaus. Our SL is subject to an applied voltage, $V(t) = V_{ac}(t) + V_{dc}$, where $V_{ac}(t) = V_{ac}\sin(2\pi f_{ac}t)$. The ac frequency $f_{ac}$ equals the golden mean times the natural frequency. Since the energy associated with $f_{ac}$ (in the MHz range) is very small compared to the typical energy scales of the system (both the energy difference between different well subbands and the level broadening due to scattering, are of the order of several meV), we can assume that the ac bias modifies adiabatically the potential profile of the SL.

We evaluate microscopically the current across barriers by means of the transfer Hamiltonian model. Electron-electron interaction is accounted for by solving the Poisson equation through the SL including the leads, with appropriate boundary conditions.[12,13,16] The condition that all voltage drops across the different regions of the nanostructure must add up to the applied bias becomes:

$$V(t) = \sum_{i=0}^{N} V_i + \sum_{i=1}^{N} V_{wi} + \frac{\Delta_1 + \Delta_2 + E_F}{e}. \qquad (1)$$

Here $V_i$, $V_{wi}$, $\Delta_1$, $\Delta_2$, and $E_F$ denote the potential drops at the $i$th barrier and well, the energy drops at emitter and collector contacts, and the collector Fermi energy, respectively. Tunneling and displacement currents are related through the Ampere's law for the total current density $J = J(t)$:

$$J = J_{i,i+1} + \frac{\epsilon}{d}\frac{dV_i}{dt}. \qquad (2)$$

Here $J_{i,i+1}$, $d$ and $\epsilon$ are the tunneling current through the $i$th barrier, its thickness, and the GaAs static permittivity, respectively.

Figure 1 shows the time-averaged current through the SL versus $V_{dc}$, the time evolution of the current at a fixed dc bias, its Fourier spectrum, and its PM for $V_{dc} = 4.2$ V and $V_{ac} = 19$ mV. This value of the dc bias corresponds to the onset of the second plateau of the time-averaged $I-V$ characteristic curve [Fig. 1(a)]. The current trace of Fig. 1(b) is quasiperiodic and does not present observable superimposed high-frequency oscillations (spikes). The natural oscillation near the onset of the plateau is caused by monopole recycling very close to the collector contact. Thus the DW does not move over many wells and no current spikes are observed. The power spectrum in Fig. 1(c) contains peaks at the fundamental frequency $f_0 \approx 39$ MHz, the frequency of the applied ac field $f_{ac}$, the combination of both and their higher harmonics. The PM depicted in Fig. 1(d) is a smooth loop with a double layer structure indicating nontrivial quasiperiodic oscillations. Let us now fix the dc bias at $V_{dc} = 5.1$ V, in the middle of the second plateau. The undriven self-oscillation is caused by recycling of monopole DWs which periodically run through part (approximately 35%) of the SL and disappear at the collector. Similar dynamics is observed with ac-driven bias. The corresponding current trace may show high-frequency spikes depending on the chosen initial field profile. A flat initial field profile gives a spiky current trace until the latter settles to the stable oscillation (without appreciable current spikes); see the earlier part of Fig. 2(a). The power spectrum and PM corresponding to this case differ markedly from those in Figs. 1(c) and 1(d). The harmonic content of the power spectrum in Fig. 2(b) (spikes present) is greater than that in Fig. 1 (spikes absent). Additional peaks correspond to higher harmonics of the natural frequency (i.e., the low fundamental frequency, $f_0 \approx 33$ MHz; the spikes have frequencies $f_s \approx 566$ MHz), the applied frequency and combinations thereof. The corresponding PM presents a distorted loop with a double layer

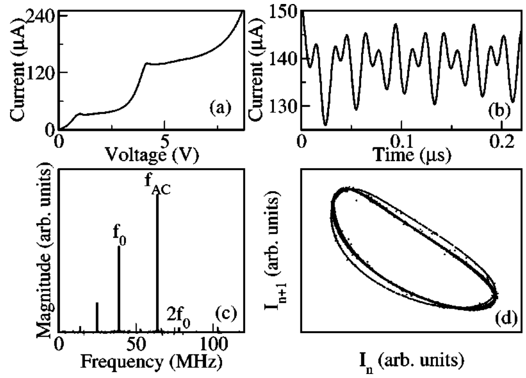


FIG. 1. (a) Time-averaged $I(V)$ curve for a 50-period 13.3-nm GaAs/2.7-nm AlAs SL with $N_w = 2\times10^{10}$ cm$^{-2}$ and $N_c = 2\times10^{18}$ cm$^{-3}$. (b) $I(t)$ for $V_{dc} = 4.2$ V, $f_0 = 39$ MHz, $V_{ac} = 19$ mV. Spikes are not resolved. (c) Power spectrum showing barely formed higher harmonics of the main frequency. (d) Poincaré map, constructed by plotting the current at the $(n+1)$st ac period versus that at the $n$th period. The times are taken at the minima of $V_{ac}(t)$.

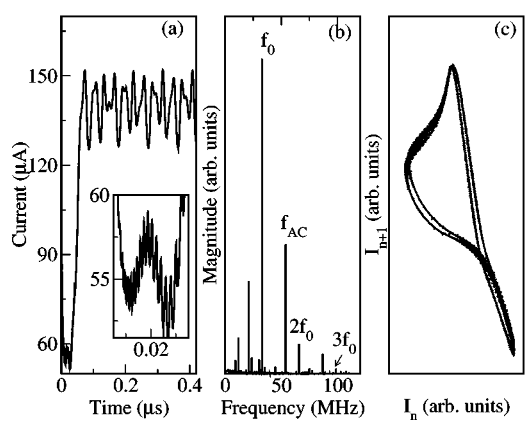


FIG. 2. (a) $I(t)$ for $V_{dc} = 5.1$ V [see Fig. 1(a)], $f_0 = 33$ MHz, $V_{ac} = 19$ mV. Spikes are present in the transient regime (see inset). (b) Power spectrum displaying several higher harmonics of $f_0$. (c) Poincaré map showing a somewhat distorted loop due to the higher harmonics in (b).

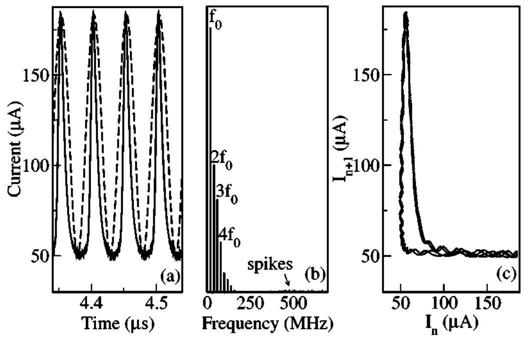


FIG. 3. (a) $I(t)$ (full line) for $V_{dc}=5.5$ V and $V_{ac}=2$ mV, $N_w=2\times10^{10}$ cm$^{-2}$, $N_c=2\times10^{16}$ cm$^{-3}$. Fitting to a sine function (dashed line) is depicted for comparison. (b) Power spectrum. (c) Poincaré map.

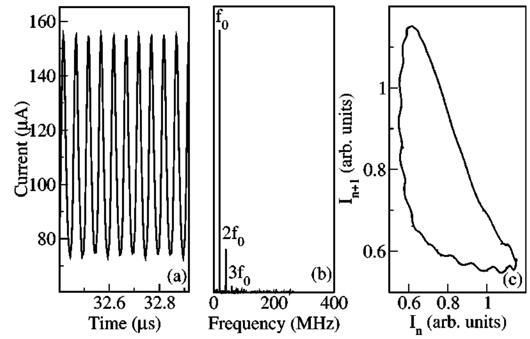


FIG. 4. (a) $I(t)$ for $V_{dc}=5.5$ V and $V_{ac}=2$ mV, $N_w=2\times10^{10}$ cm$^{-2}$, $N_c=4\times10^{16}$ cm$^{-3}$. (b) Power spectrum. (c) Poincaré map.

structure which shows a strong similarity with experimental results;[6,7] see Fig. 2(c). From these numerical observations, we conclude that distorted PM are linked to spiky current traces, even if such traces change to smooth ones after a transient.[20]

In Ref. 7, higher harmonics observed in real-time traces of the current are held responsible for the occurrence of distorted PM in the quasiperiodic case. Further experimental evidence is found in the frequency-locked state. Support for this claim lies on numerical studies of power spectrum data. The PM plot of a signal containing both the natural frequency and the ac field oscillations is a perfect ellipse. Adding a new signal with the second harmonic frequency results in a PM with a twisted loop. Adding more and more harmonics may reproduce reasonably well the experimental map. Thus any nonsinusoidal natural periodic signal in the presence of a weak ac potential of incommensurate frequency seemingly produces a distorted PM. The corresponding Fourier spectrum presents peaks at the main natural frequency and its higher harmonics. To observe self-crossings in the first return map, peaks at higher harmonics in the power spectrum should be large enough for the current trace to acquire at least one additional maximum. What is the physical origin of these additional peaks? We argue that they correspond to current spikes.

Figure 3(a) shows the current through a 50-well 13.3 nm/2.7 nm SL driven by both a dc voltage bias (5.5 V) and an ac bias having 2 mV amplitude and frequency equal to the golden mean ratio times the natural frequency (about 20 MHz). Doping in the wells and in the contacts are $N_w=2\times10^{10}$ cm$^{-2}$ and $N_c=2\times10^{16}$ cm$^{-3}$, respectively (low contact doping is needed to obtain sharper monopoles). The current trace deviates from a sine due to the presence of spikes at low current values, which results in higher harmonics in the frequency spectrum [Fig. 3(b)]. The first return map is strongly distorted [see Fig. 3(c)]: its twisted arm extends from about 49 $\mu$A to 55 $\mu$A, exactly the region covered by the spikes in Fig. 3(a). Let us see that current spikes ultimately cause the twist of the PM loop. If we slightly increase contact doping, $N_c=4\times10^{16}$ cm$^{-3}$, both the distortion in the current PM and the spikes vanish [Fig. 4(a)]. Although the power spectrum shows higher harmonics [Fig. 4(b)], the corresponding first-return map [Fig. 4(c)] is smooth without self-crossings.

To reinforce the previous conclusions, we change the dc voltage to $V_{dc}=1.5$ V (middle of the first plateau, 4 MHz natural frequency) and $V_{ac}=2$ mV, so that undriven self-oscillations are caused by dipole waves, yielding spikier current traces.[16] Figure 5(a) depicts the current trace and its inset shows superimposed current spikes. The corresponding PM presents three well-defined distorted loops, as seen in Fig. 5(c). Since loops in the PM are due to a combination of strong enough signals of different frequencies,[7] the greater strength of the high-frequency spikes gives rise to higher harmonic content and additional loop structure [Fig. 5(b)].

As mentioned above, the double-layer structure of PM indicates nonchaotic quasiperiodic attractors having fractal phase space structures. We have calculated the multifractal dimensions of these attractors, $D_q$, for three cases corresponding to Figs. 1, 2, and 5; see Fig. 6. In all cases, they correspond to strange attractors whose $D_q$ presents the kneelike structure typical of chaotic attractors with multifractal dimensions.[9] Albeit they are not chaotic, it is important to stress that strangeness means fractal phase space structure and does not involve necessarily sensitive dependence on

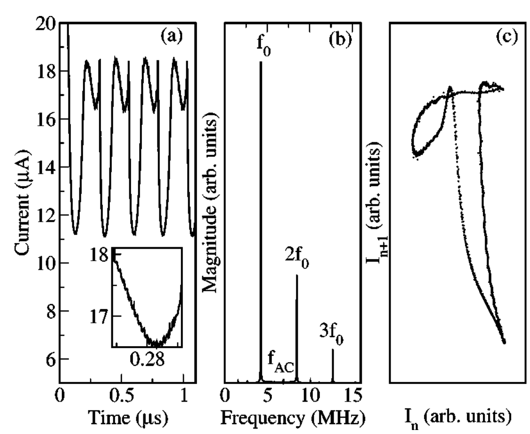


FIG. 5. (a) $I(t)$ for $V_{dc}=1.5$ V, $f_0=4$ MHz, $V_{ac}=2$ mV, $N_w=2\times10^{10}$ cm$^{-2}$, $N_c=2\times10^{16}$ cm$^{-3}$. Spikes are superimposed on the current throughout the signal (see inset). (b) Power spectrum. Higher harmonics of $f_0$ have finite amplitude. (c) Poincaré map.

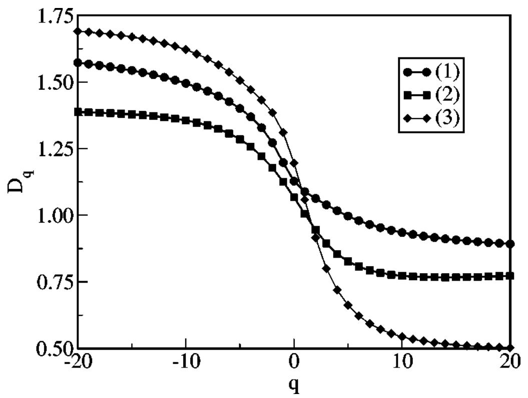


FIG. 6. Multifractal dimension. Labels (1), (2), and (3) correspond to the attractors in 1(c), 2(c), and 5(c), respectively.

initial conditions (as for chaotic attractors). This distinction is well established both theoretically,[18,19] and experimentally.[21]

In summary, we have explained recent experimental evidence showing complex Poincaré maps in the quasiperiodic regime.[7] We have analyzed theoretically the time-dependent current through an ac driven SL and have characterized intriguing Poincaré maps corresponding to quasiperiodic oscillations. The strange attractors which define them have their physical origin in the complex dynamics of the domain wall. We have shown that distorted loops in the Poincaré maps are related to the presence of high-frequency spikes in the current traces. Their frequencies combine with the ac frequency and the low natural frequency to yield a richer power spectrum. Since spikes are associated to extended motion of the DW, distorted Poincaré maps are observed at the middle of a $I-V$ plateau and not at its beginning, where the monopole moves over too small a part of the SL. The case of natural self-oscillations due to dipole recycling is different: dipole DW are generated at the emitter contact and move over the whole SL. Then spikes are more prominent than in the monopole case, yielding current traces with higher harmonic content and distorted PM. Thus our model shows that higher frequency current spikes are responsible for new features in the time traces of the electronic current when driven by an ac potential. Hopefully our results may help understanding dynamical properties of doped semiconductor superlattices better.

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