Temperature-induced breakdown of stationary electric field domains in superlattices

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Abstract

We have analyzed the effect of temperature on the current–voltage characteristics of a doped weakly coupled superlattice by means of a discrete drift-diffusion model including the Coulomb interaction self-consistently. At low temperature, stationary electric field domains coexist in the sample. Increasing the temperature destroys this stable electric field configuration and there appear self-sustained oscillations of the current, in agreement with experimental evidence. Our theoretical model explains how the self-oscillation frequency and amplitude depend on the external DC voltage. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

Stationary electric field domains appear in voltage-biased weakly coupled superlattices (SLs) for large enough doping [1–3]. When the carrier density is below a critical value, current self-oscillations may appear in appropriate voltage windows that widen and coalesce as doping is further decreased. Below a critical doping value, the total current density ($J$) becomes again stationary and the corresponding electric field is almost uniform inside the sample [4]. Self-oscillations are caused by the periodic recycling and motion of a domain wall (DW) separating two different electric field domains inside the structure. The DW is formed by a charged accumulation layer (also dubbed monopole). Clearly doping is not a feasible control parameter, so that the other quantities affecting carrier density should be used instead to observe these phenomena. Laser illumination in undoped SLs [5–7], or transverse magnetic fields [8], and temperature ($T$) [9–12] in doped SLs have shown to be good alternative control parameters. Recently, intense THz radiation applied to n-doped multiple quantum wells has been also proven to induce a similar transition from stationary electric field domains to current self-oscillations [13].

The governing equations describing charge transport in a weakly coupled SL with $N$ wells are Ampère’s and Poisson’s:

$$\frac{dF_i}{dt} + \frac{en_i(F_i)}{L} - eD(F_i) \frac{n_{i+1} - n_i}{L^2} = J(t),$$

(1)

$$F_i - F_{i-1} = \frac{e}{\varepsilon} (n_i - N_D^w)$$

(2)
for the variables \( J, F_i \) and \( n_i \). \( F_i \) is minus an average electric field on a SL period comprising the \( i \)th well and the \( i \)th barrier (well \( i \) lies between barriers \( i-1 \) and \( i \); barriers 0 and \( N \) separate the SL from the emitter and collector contact regions, respectively). \( n_i \) is the electron density at well \( i \), which is singularly concentrated on a plane located at the end of the well. In Eqs. (1) and (2), \( e, \epsilon, N_w^D, L = d + w, d \) and \( w \) are sample permittivity, minus the electron charge, 2D well doping, SL period, barrier width and well width, respectively. The Ampère law (1) establishes that the current density is the sum of displacement and tunneling currents. The latter consists of a drift term, a diffusion term, and a drift term, \( eD(F_i)(n_{i+1} - n_i)/L^2 \), calculated from a sequential tunneling microscopic description. Briefly, the tunneling current (\( J \)) through the \( i \)th barrier is a function of the densities at adjacent wells, \( n_i \) and \( n_{i+1} \), and of the potential drops at barriers \( i \) and \( i \pm 1 \), including appropriate Fermi factors [14]. Over a wide temperature range, the tunneling current is linear in \( n_i \) and \( n_{i+1} \). To calculate the drift velocity and electron diffusivity, we assume that the voltage drop \( eFL \) is the same within a SL period. Then \( J \) is a function of \( n_i, n_{i+1}, F \) and \( T \); and we use the formulas \( v(F) = L \frac{J}{N(D)}(F) \) and \( D(F) = -L^2 \frac{J}{N(D)\frac{F}{T}}\), for each value of \( T \) [14].

Equations (1) and (2) hold for \( i=1, \ldots , N-1 \) and for \( i = 1, \ldots , N \), respectively. The diffusion coefficient is a rapidly decreasing function of electric field, which implies we can safely set \( D \equiv 0 \) in our model equations for the experimentally observed voltage range except in the first plateau (low voltages) of the current–voltage characteristic. To completely specify the solution for \( D \equiv 0 \), we need to give an initial field profile, one boundary condition for \( F_0 \) (the field at the contact region) and the DC voltage bias condition, \( L \sum_{i=1}^{N} F_i = V \). The condition \( n_1 = (1+c)N_D^0 \) (excess electron density in the first well due to tunneling from the highly doped contact region [9]) and (2) specify \( F_0 \). For a more realistic condition yielding the same behavior, see Refs. [2,14].

2. Results and comparison with experiments

We have calculated at different temperatures (ranging from 110 to 150 K) the time-averaged current–voltage characteristic curve at the second plateau of a GaAs/AlAs SL. We have also obtained the time-dependent current at a fixed voltage \( V = 2.94 \) V for different temperatures. Curves for \( T \neq 110 \) K are offset for clarity.

![Fig. 1. (a) J/V curve for different temperatures, showing stationary (dynamic) states with full (empty) circles. The curve corresponding to 150 K has been shifted by \(-0.04\) mA for clarity. Lines are plotted only for eye-guiding purposes. (b) Time-dependent current at a fixed voltage \( V = 2.94 \) V for different temperatures. Curves for \( T \neq 110 \) K are offset for clarity.](image-url)
2.8 2.82 2.84 2.86 2.88 2.9 2.92
Voltagge (V)

0 10 20 30 40 50 60 70
Amplitude (µA)

and shrink as \( T \) increases. Fig. 1(b) shows that (at \( V = 2.94 \) V) \( J \) undergoes self-oscillations whose amplitude increases with \( T \) and is stationary for \( T < 115 \) K. In an interval of self-oscillations, their frequency first increases with voltage, it reaches a maximum and then drops to a smaller value than the initial one at the upper end of the interval (not shown here) [15]. Fig. 2 shows the amplitude of the self-oscillations as a function of \( V \). The amplitude vanishes at the upper and lower ends of each voltage interval. This suggests that self-oscillation branches begin and end at supercritical Hopf bifurcations [4]. As \( T \) increases, the region of negative differential mobility in the drift velocity curve \( v(F) \) shrinks and the average frequency and maximum amplitude of the self-oscillations increase in each voltage interval [15]. (see also Fig. 2) The situation we have described resembles that obtained as voltage and doping are varied at fixed low temperature, provided doping and reciprocal of temperature are assimilated. In Ref. [4] the phase diagram doping-voltage of a doped SL was calculated. At low doping (high \( T \)), the electric field inside the SL is almost homogeneous and stationary. Above a critical value, branches of self-oscillations appear. In this region, there are voltage intervals of stationary electric field profiles separated by intervals of self-oscillations. The latter arise and disappear (typically) as Hopf bifurcations from stationary states. Above a certain doping (low \( T \)), the intervals of self-oscillations vanish and only stationary states (consisting of two electric field domains separated by a DW) remain.

Fig. 3 shows the time evolution of the carrier density at four DC biases. Its peak measures the DW location. During one cycle of the oscillation the DW is near the end of the SL, it moves through a few wells, and vanishes at the collector. At the same time, the emitter launches a new monopole and a new current period starts. The insets of Fig. 3 show that the maximum current during an oscillation period does not change very much as the voltage increases. However, the minimum current behaves differently: it drops very rapidly for larger voltages, which indicates an increase of the amplitude with voltage. This behavior can be explained in the following way. The monopole is formed when the current is greater than that corresponding to the local maximum of the velocity curve, \( eN_D^wv_{\text{max}}/L \). As \( J \) approximately follows the velocity curve, the current decreases with time until either the DW vanishes in the collector or the minimum value \( eN_D^wv_{\text{min}}/L \) is attained. The former situation occurs at low voltages while the latter takes place at high \( V \) (because in this case the current has enough room to follow the whole velocity curve). The branch of current self-oscillations starts at zero amplitude via a supercritical Hopf bifurcation, and \( J \) does not depart too much from \( eN_D^wv_{\text{max}}/L \). For larger voltages, the
minimum current during one oscillation period reaches lower values.

3. Conclusions

In summary, we have studied the second plateau of the $J/V$ curve corresponding to a weakly coupled SL at different temperatures by means of a discrete drift-diffusion model self-consistently including Coulomb interaction and boundary conditions. In agreement with available experimental data [10–12], we observe that increasing $T$ facilitates current self-oscillations and oscillatory current windows are opened in the $J/V$ characteristic curve. In the ends of these windows, the oscillation amplitude is zero while its frequency is not, which suggests oscillation branches start and end at Hopf bifurcations. All these features are qualitatively explained.

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References