Nonlinear Dynamics in Far-infrared-driven Quantum Wells

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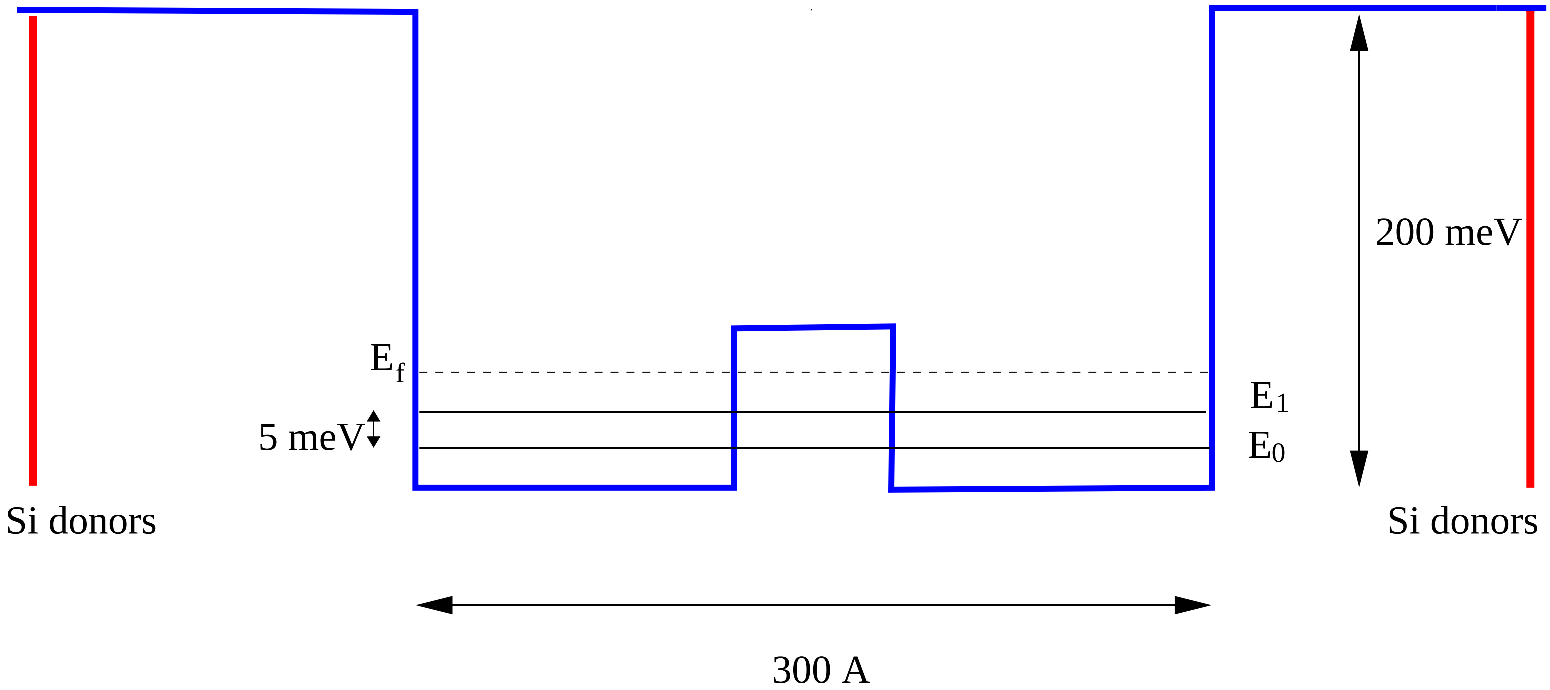
http://www.vjnano.org OPTICAL PROPERTIES AND QUANTUM OPTICS.



Possible application for optical switching

Realization of optical logic gates

Operation in the THz range

Intrinsic Optical Bistability

Nonlinear dynamics effect in a quantum system

The Hamiltonian

$$\hat{H} = \hat{H}_0 + \hat{H}_{el} + \hat{H}_{el-b} + H_b$$

Bare quantum well term:

$$\hat{H}_0 = \int d^3x \hat{\psi}^\dagger(x)(-\frac{\hbar^2}{2m^*}\nabla^2 + W(z))\hat{\psi}(x)$$

The electron-electron interaction term :

$$\hat{H}_{el-el} = \frac{e^2}{2}\int d^3x \int d^3x' \hat{\psi}^\dagger(x)\hat{\psi}^\dagger(x') \frac{e^{-\mu|x-x'|}}{\kappa|x-x'|}\hat{\psi}(x')\hat{\psi}(x).$$

Electron-background interaction term :

$$\hat{H}_{el-b} = -e^2 \int d^3x d^3x' \hat{\psi}^\dagger(x') n(x) \frac{e^{-\mu|x-x'|}}{\kappa|x-x'|}\hat{\psi}(x')$$

and the background contribution :

$$H_b = \frac{e^2}{2}\int d^3x \int d^3x' \frac{n(x)n(x')e^{-\mu|x-x'|}}{\kappa|x-x'|}.$$

The Heisenberg equations of motion

$$i\hbar \frac{\partial \hat{\psi}}{\partial t} = [\hat{\psi}, \hat{H}]$$

The operators obey the usual Fermi anti-commutation relations

$$\{\hat{\psi}^\dagger(x), \hat{\psi}(x')\} = \delta^3(x - x')$$

$$i\hbar \frac{\partial \hat{\psi}}{\partial t}(x,t) = \left[-\frac{\hbar^2}{2m^*}\nabla^2 + W(z) \right] \hat{\psi}(x,t) - e^2 \int dx' n_D(x') \frac{e^{-\mu|x-x'|}}{\kappa|x-x'|} \hat{\psi}(x,t)$$
$$+e^2 \int dx' \frac{e^{-\mu|x-x'|}}{\kappa|x-x'|} \hat{\psi}^\dagger(x',t)\hat{\psi}(x',t)\hat{\psi}(x,t),$$

$n_D(z) = N_s/2[\delta(z-L) + \delta(z+L)]$, where N_s is the sheet density.

$$\hat{\psi}(x,t) \equiv \sum_{n,k} \frac{e^{ik\cdot\rho}}{\sqrt{A}} \xi_n(z) \hat{a}_n(k,t),$$

The elimination of divergences

$$\int dx' n_D(x') V_\mu(x - x') = \frac{\pi e^2 N_s}{\kappa\mu}(e^{-\mu|z-L|} + e^{-\mu|z+L|}),$$

$$\int dx' V_\mu(x - x')\hat{\psi}^\dagger(x', t)\hat{\psi}(x', t) = \frac{2\pi e^2}{\kappa A} \sum_{n_1 n_2, k_1 k_2} a^\dagger_{n_1 k_1} a_{n_2 k_2} e^{-i\rho\cdot(k_1 - k_2)} \times$$

$$\int dz' \xi_{n_1}(z') \xi_{n_2}(z') \int_0^\infty r J_0(r|k_1 - k_2|) \frac{e^{-\mu\sqrt{r^2+(z-z')^2}}}{\sqrt{r^2 + (z - z')^2}} dr.$$

The $k_1 = k_2$ term in the above sum can be simplified to

$$\frac{2\pi e^2}{\kappa\mu A} \sum_{n_1 n_2, k} a^\dagger_{n_1 k} a_{n_2 k} \int dz' \xi_{n_1}(z') \xi_{n_2}(z') e^{-\mu|z-z'|} =$$

$$\frac{2\pi e^2}{\kappa\mu A} \left[\hat{N} - \mu \sum_{n_1 n_2, k} a^\dagger_{n_1 k} a_{n_2 k} \int dz' \xi_{n_1}(z') \xi_{n_2}(z') |z - z'| + O(\mu^2) \right].$$

The diverging terms above cancel out and we can take the limit $\mu \to 0$ obtaining the depolarization-shift terms in the Heisenberg equation.

The Hartree Approximation

$$
\begin{aligned}
i\hbar \frac{\partial \hat{\psi}}{\partial t}(x,t) &= \left[-\frac{1}{2m} \hbar^2 \nabla^2 + W(z) \right] \hat{\psi}(x,t) \\
&\quad - \frac{2\pi e^2}{\kappa A} \sum_{n_1 n_2, k} a^\dagger_{n_1 k} a_{n_2 k} \int dz' \xi_{n_1}(z') \xi_{n_2}(z') |z - z'| \hat{\psi}(x,t) \\
&\quad + \frac{2\pi e^2}{\kappa A} \sum_{n_1 n_2, k_1 \neq k_2} a^\dagger_{n_1 k_1} a_{n_2 k_2} e^{-i\rho \cdot (k_1 - k_2)} \times \\
&\quad \int dz' \xi_{n_1}(z') \xi_{n_2}(z') \frac{e^{-|k_1 - k_2||z - z'|}}{|k_1 - k_2|} \hat{\psi}(x,t).
\end{aligned}
$$

After the Hartree approximation is applied, we obtain

$$
\begin{aligned}
i\hbar \frac{\partial \hat{\psi}}{\partial t}(x,t) &= \left[-\frac{1}{2m^*} \hbar^2 \nabla^2 + W(z) \right] \hat{\psi}(x,t) \\
&\quad - \frac{2\pi e^2}{\kappa A} \sum_{n_1, k} \langle a^\dagger_{n_1 k} a_{n_2 k} \rangle \int dz' \xi_{n_1}(z') \xi_{n_1}(z') |z - z'| \hat{\psi}(x,t).
\end{aligned}
$$

The envelope wavefunctions

If $a_n(k,t) = a_n(k)e^{i\epsilon_n(k)t}$, then $\xi_n(z)$ obeys the Schrödinger equation for the nth subband wave function given by

$$\left[-\frac{1}{2m^*}\frac{d^2}{dz^2} + W(z) + V_H(z)\right]\xi_n(z) = \epsilon_n \xi_n(z).$$

The Hartree potential $V_H(z)$ is given by

$$V_H(z) = \frac{-2\pi e^2}{\kappa}\int_{-\infty}^{\infty} dz' |z - z'| n(z').$$

The electron number density is given by

$$n(z) = \langle \hat{\psi}^\dagger(z)\hat{\psi}(z)\rangle = \frac{1}{A}\sum_{n,k} |\xi_n(z)|^2 f_n(k)$$

where $f_n(k)$ is the Fermi occupation function.

The Time-Dependent Hartree Approximation

Including the external field, the Heisenberg equation in the time-dependent Hartree approximation becomes

$$
\begin{aligned}
i\hbar\dot{a}_{n'p} \quad &= \quad \epsilon_n(p)a_{n'p} - \sum_{n_1 n_2, n_3, k} \left[\langle a^\dagger_{n_1 k} a_{n_2 k} \rangle - f_{n_1}(k)\delta_{n_1, n_2} \right] \times \\
& \qquad V^0_{n' n_2 n_3 n_4} a_{n_3 p} - \sum_m F(t)\mu_{n' m} a_{mp}.
\end{aligned}
$$

where

$$
V^0_{n_1 n_2 n_3 n_4} = \frac{2\pi e^2}{\kappa A} \int dz \int dz' \xi_{n_1}(z)\xi_{n_2}(z')|z - z'|\xi_{n_3}(z')\xi_{n_4}(z),
$$

and $\mu_{nm} = e \int dz\, \xi_n(z) z \xi_m(z)$.
We will write the equations of motion for the 1-particle density matrix

$$
\sigma_{n,n'}(k) =< a^\dagger_{nk} a_{n'k} >
$$

so we can introduce dissipation.

The density-matrix equations of motion

$$
\begin{aligned}
-i\hbar\frac{\partial}{\partial t}\sigma_{n,n'} \quad = \quad & (\epsilon_n - \epsilon_{n'})\sigma_{n,n'} + i\hbar\gamma_{nn'}(\sigma_{n,n'} - \sigma^0_{nn'}) \\
& + AN_s \sum_{n_2 n_3 n_4} V^0_{n' n_2 n_3 n_4}\sigma_{n,n_4}(\sigma_{n_2 n_3} - \sigma^0_{n_2 n_3}) \\
& - AN_s \sum_{n_1 n_2 n_3} V^0_{n_1 n_2 n_3 n}\sigma_{n_1,n'}(\sigma_{n_2 n_3} - \sigma^0_{n_2 n_3}) \\
& - \sum_m \mathcal{E}(t)\Big(\mu_{mn}\sigma_{mn'} - \mu_{n'm}\sigma_{nm}\Big)
\end{aligned}
$$

where $\sigma_{n,n'} = \frac{1}{AN_s}\sum_k \sigma_{n,n'}(k)$ and $\sigma^0_{nn'}$ is the equilibrium density matrix.
Two-Subband case:

$$
\begin{aligned}
\dot{\Delta} \quad &= \quad -\gamma_1(\Delta - \Delta_0) + 4Im\sigma_{10}V_{10}(t) \\
\dot{\sigma}_{10} \quad &= \quad i\epsilon_{10}\sigma_{10} - \gamma_2\sigma_{10} - i\Delta V_{10}(t) \\
& \quad\quad -i\sigma_{10}[\alpha\Big(\zeta Re\sigma_{10} + \beta(\Delta - \Delta_0)/4\Big) + \mathcal{E}(t)(\mu_{11} - \mu_{00})].
\end{aligned}
$$

Where $V_{10}(t) = \mu_{10}\mathcal{E}(t) + \alpha\Big[Re\sigma_{10} - \zeta(\Delta - \Delta_0)/4\Big]$ and α, ζ, and β are constants

The Averaging Method

Suppose we have a differential equation

$$\dot{x} = \epsilon f(x,t), \qquad x \in \mathbf{C}^n, \qquad 0 < \epsilon \ll 1, \tag{1}$$

with $f(x,t)$ T-periodic and sufficiently well behaved (C^2 is enough). The function f can be decomposed in its Fourier modes, $f(x) = f_0(x) + \tilde{f}(x,t)$, where $f_0(x)$ has no explicit time dependence and $\tilde{f}(x,t)$ includes all the oscillating terms. The averaging theorem states that in the limit $\epsilon \to 0$ Eq. (1), through the transformation $x = y + \epsilon w(y,t)$, can be replaced by

$$\dot{y} = \epsilon f_0(y) + \epsilon^2 D\tilde{f}(y,t) w(y,t).$$

The function $w(y,t)$ is chosen to satisfy the differential equation $w_t(y,t) = \tilde{f}(y,t)$ with integration constants set to zero. Since here we are interested only in first-order averaging we neglect terms $O(\epsilon^2)$. We are then left with

$$\dot{y} = \epsilon f_0(y). \tag{2}$$

This replacement means that for the same initial conditions solutions to Eqs. (1) and (2) will be close to order $O(\epsilon)$ in a time scale of $O(1/\epsilon)$.

Averaged density-matrix equations

If we assume almost equally spaced levels we can put the equations of motion for $\sigma_{n,n'}$ in slowly varying form with the transformation $\sigma_{n,n'} - \sigma^0_{n,n'} = e^{(i(n-n')\omega t}\rho_{nn'}$

The applied external field is given by $\mathcal{E}(t) = \mathcal{E}_0 e^{i\Omega t} + \mathcal{E}_0^* e^{-i\Omega t}$ where $\Omega = N\omega$, N integer.
After averaging is performed we obtain

$$-i\frac{\partial}{\partial t}\rho_{n,n'} = (\delta_{nn'} - i\gamma_{nn'})\rho_{n,n'}$$

$$-\sum_{n_2 n_3 n_4} \tilde{V}^0_{n' n_2 n_3 n_4}\left(\rho_{n,n_4}\delta_{n'+n_2,n_3+n_4} + \sigma^0_{nn_4}\delta_{n'+n_2,n_3+n}\right)\rho_{n_2 n_3}$$

$$+\sum_{n_1 n_2 n_3} \tilde{V}^0_{n n_2 n_3 n_4}\left(\rho_{n_4,n'}\delta_{n_2+n_4,n+n_3} + \sigma^0_{n_4 n'}\delta_{n'+n_2,n+n_3}\right)\rho_{n_2 n_3}$$

$$-\sum_m \Big[\tilde{\mu}_{mn}\rho_{mn'}(\mathcal{E}_0\delta_{m-n,N} + \mathcal{E}_0^*\delta_{m-n,-N})$$

$$-\tilde{\mu}_{n'm}\rho_{nm}(\mathcal{E}_0\delta_{n'-m,N} + \mathcal{E}_0^*\delta_{n'-m,-N})\Big]$$

$$-(\mathcal{E}_0\delta_{n-n',N} + \mathcal{E}_0^*\delta_{n-n',-N})\mu_{n'n}(\sigma^0_{n'n'} - \sigma^0_{nn}).$$

Optical Bistability and Second-Harmonic Generation

Averaged two-subband density matrix equations

$$\begin{aligned} \dot{\Delta} &= -\gamma_1(\Delta - \Delta_0) - i\lambda(\sigma - \sigma^*), \\ \dot{\sigma} &= (i\delta - \gamma_2)\sigma - i\frac{\alpha\Delta}{2}\sigma - i\alpha\sigma\beta(\Delta - \Delta_0)/4 - i\lambda\Delta/2, \end{aligned}$$

where $\delta = \tilde{\epsilon}_{10} - 1$ is the detuning and $\lambda = \mathcal{E}_0$. The equation for the fixed point is a cubic. Optical bistability occurs when we have three real roots of the cubic.

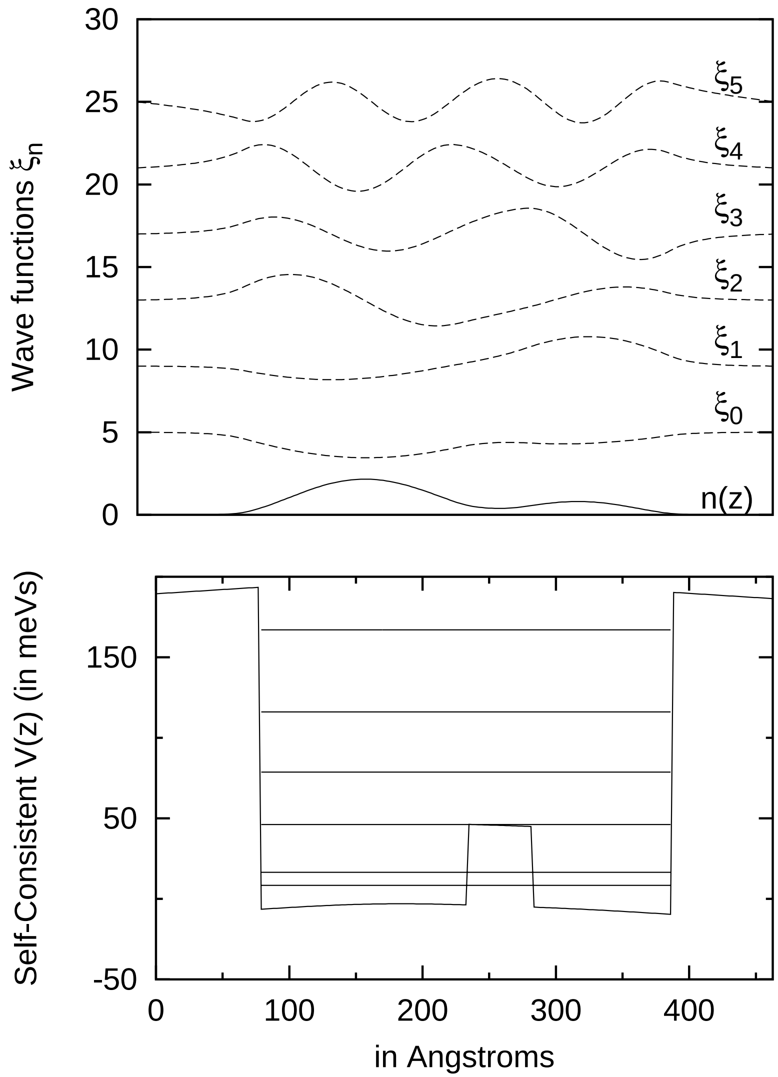
The absorption coefficient in the quantum well is given by

$$G = \frac{1}{T}\int_0^T dt\, eE(t)\frac{d}{dt}Tr\left[\sigma(t)z\right],$$

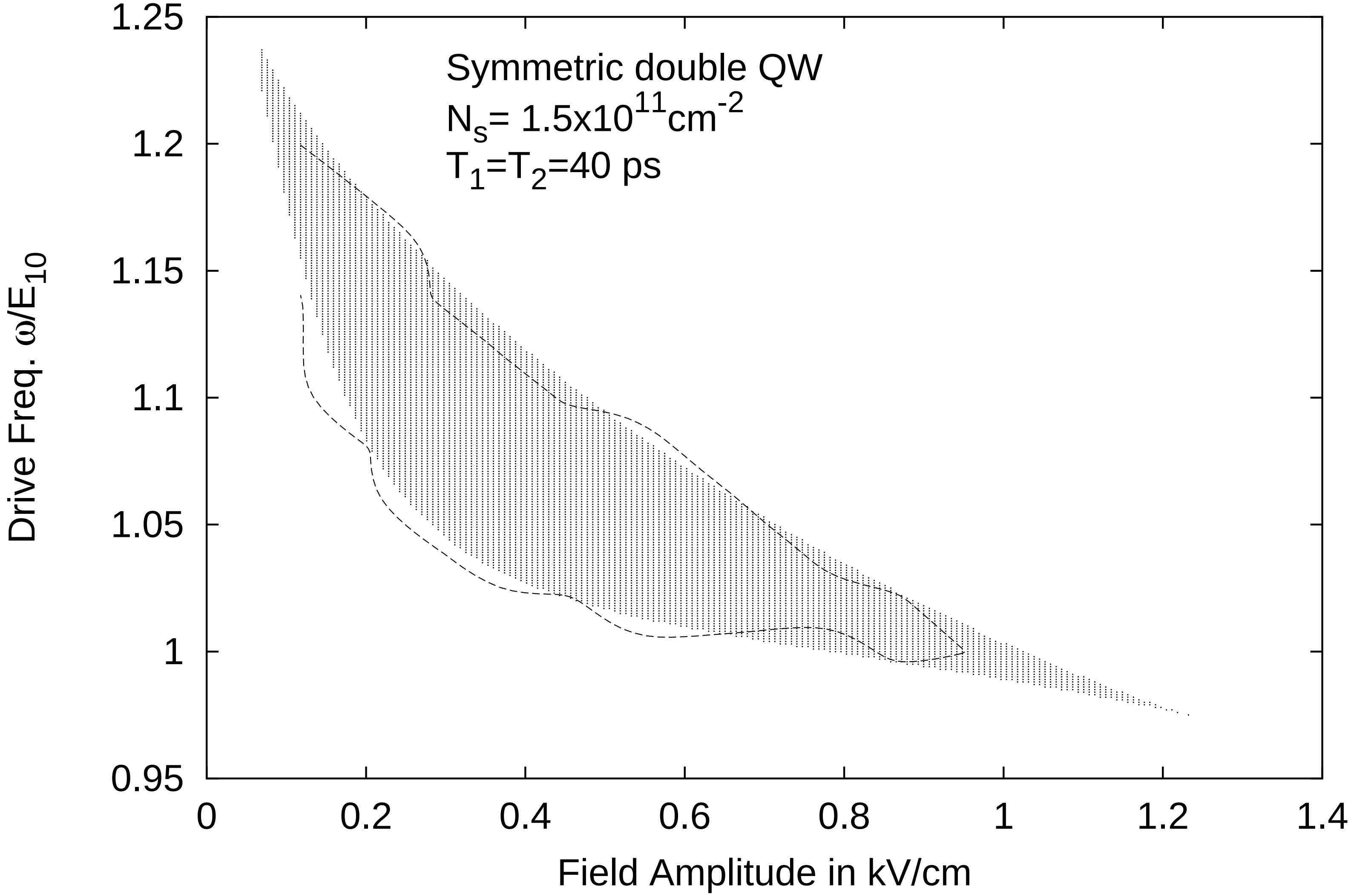
For the many-subband case when the subband energies are almost equally spaced we obtain

$$G = -2e\omega \sum_{n>m}(n-m)(\mathcal{E}_0\delta_{n-m,-N} + \mathcal{E}_0^*\delta_{n-m,N})\tilde{\mu}_{mn}\mathrm{Im}\rho_{nm}.$$

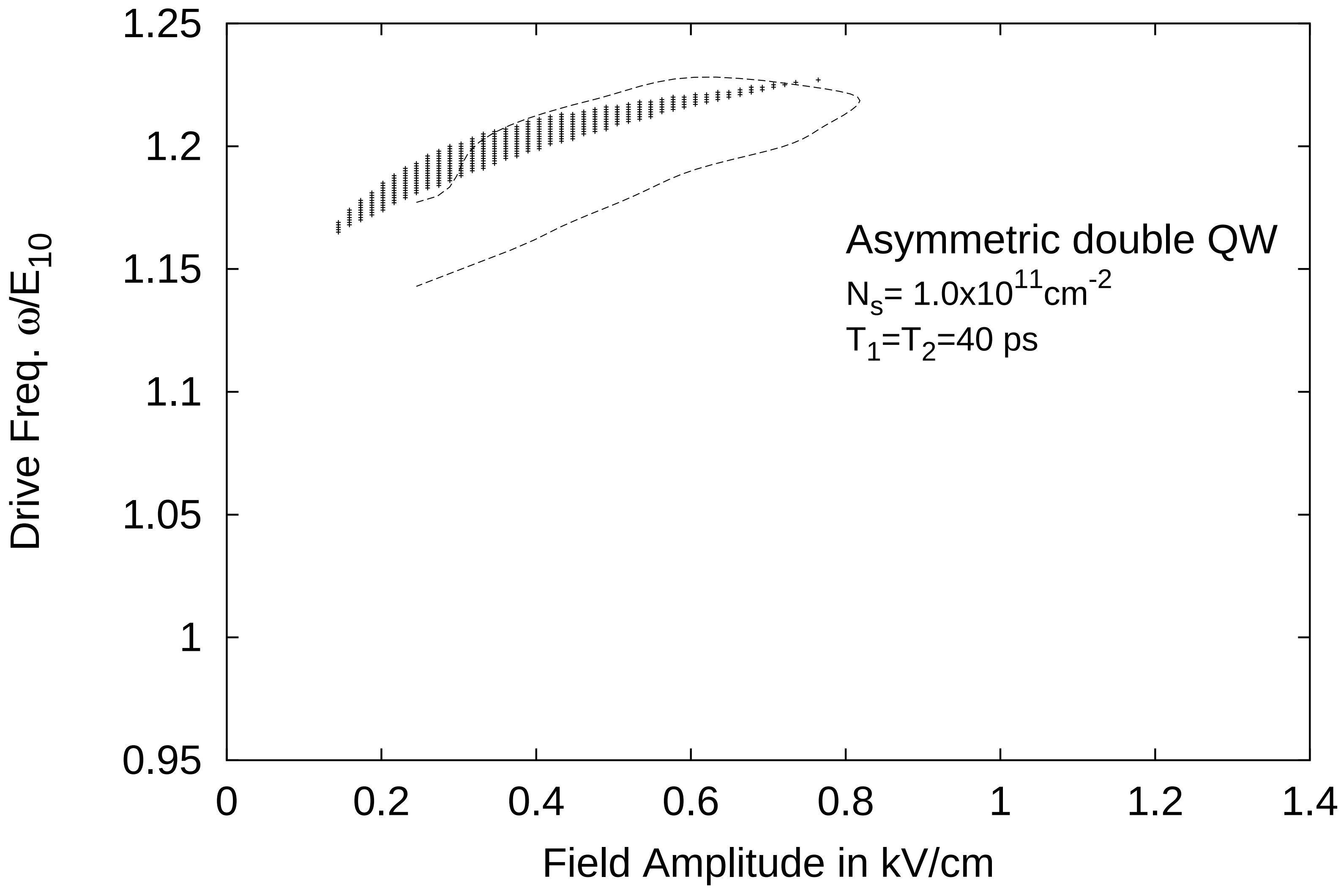
The Asymmetric QW, $N_s = 1.5 \times 10^{11} \mathrm{cm}^{-2}$



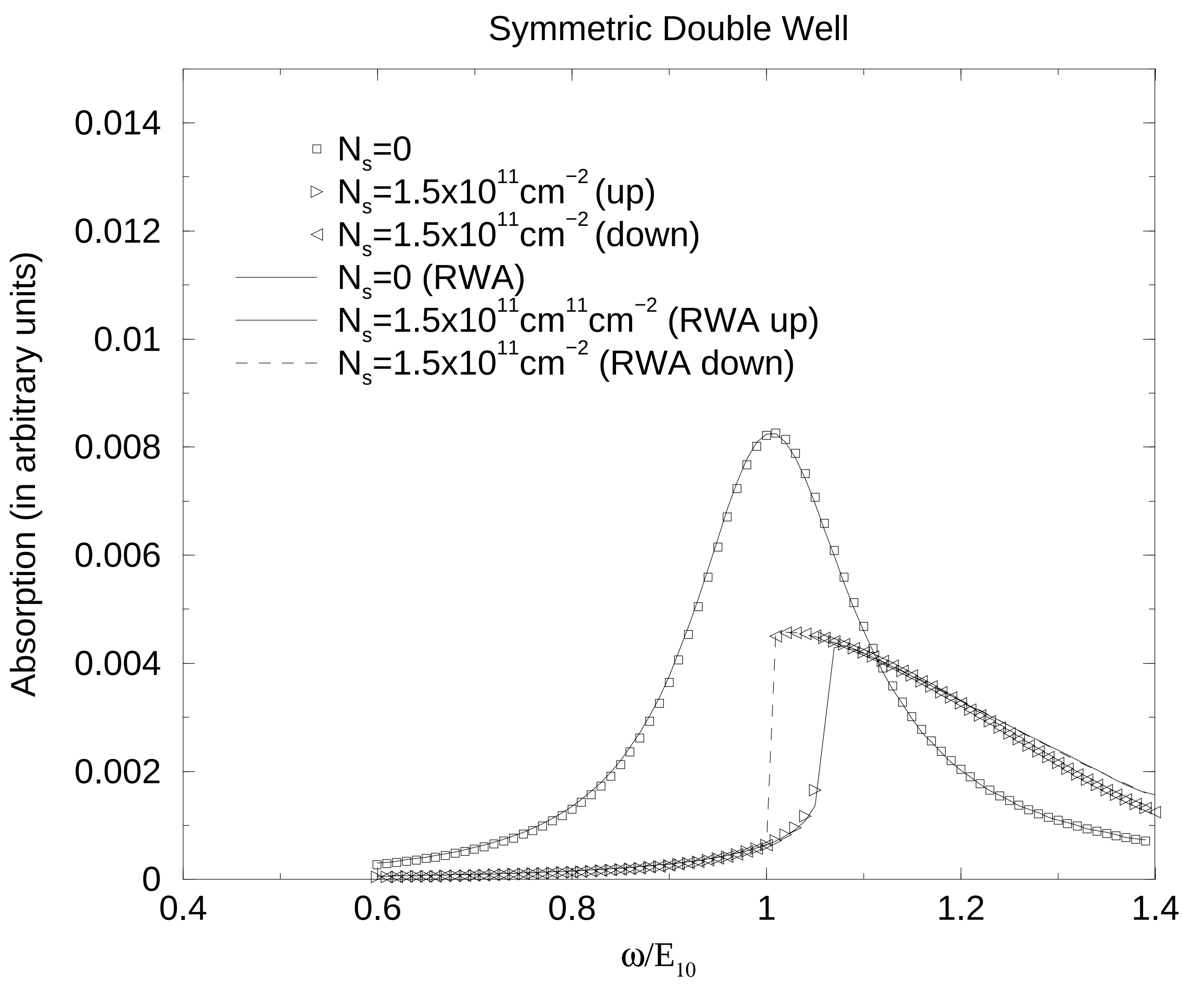
Range of Optical bistability activity in a symmetric QW



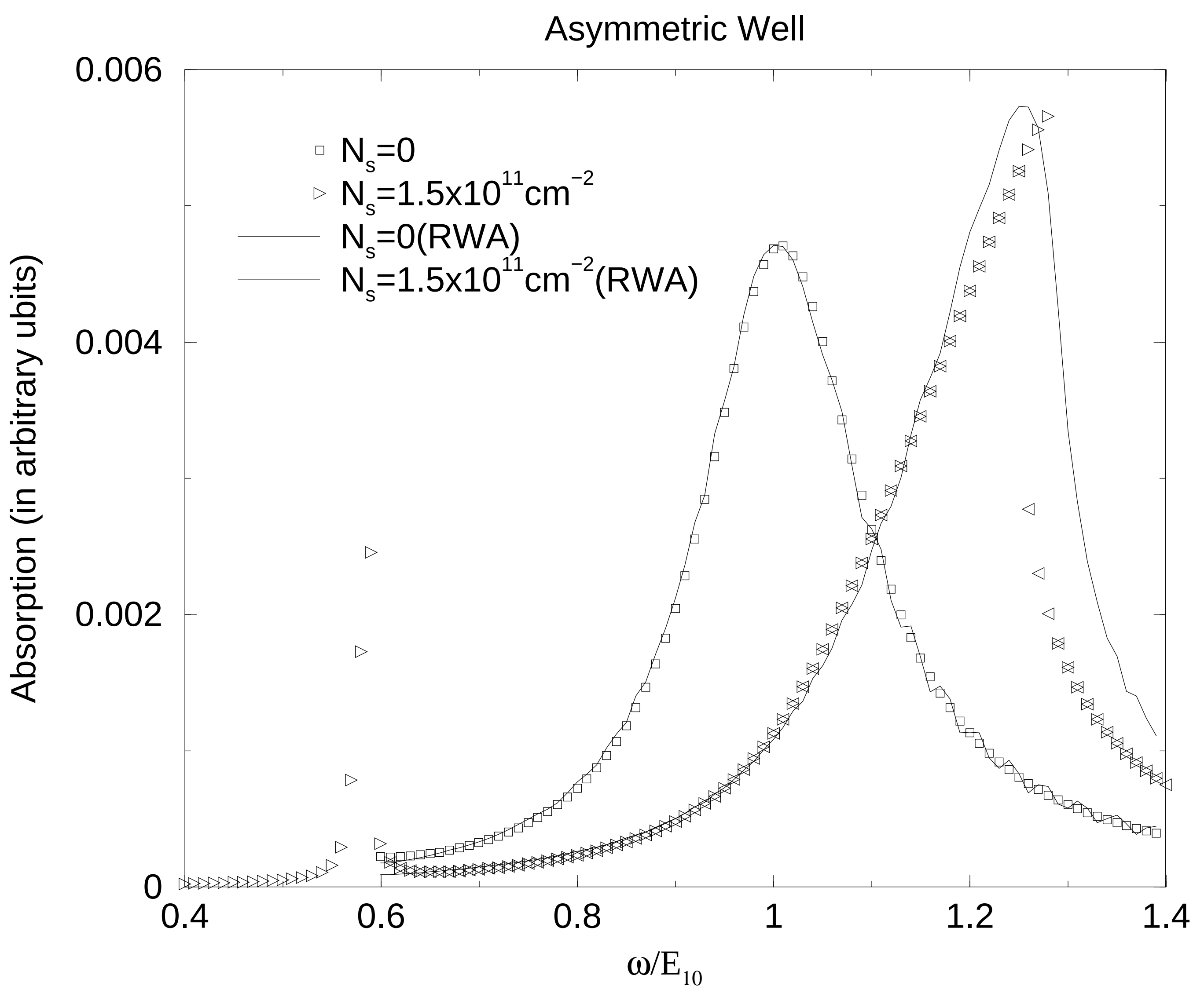
Range of Optical bistability activity in an asymmetric QW

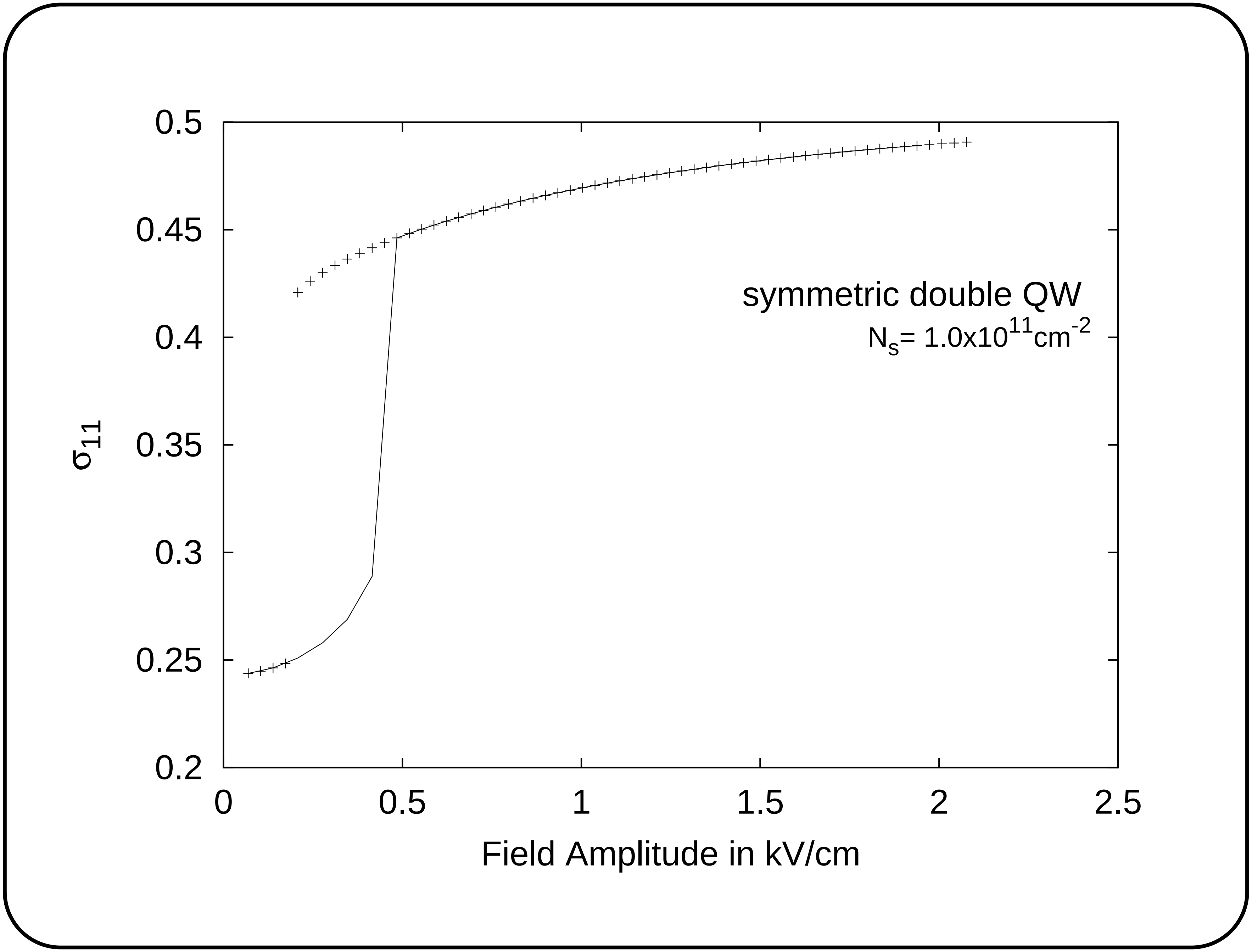


Absorption lines in the symmetric QW

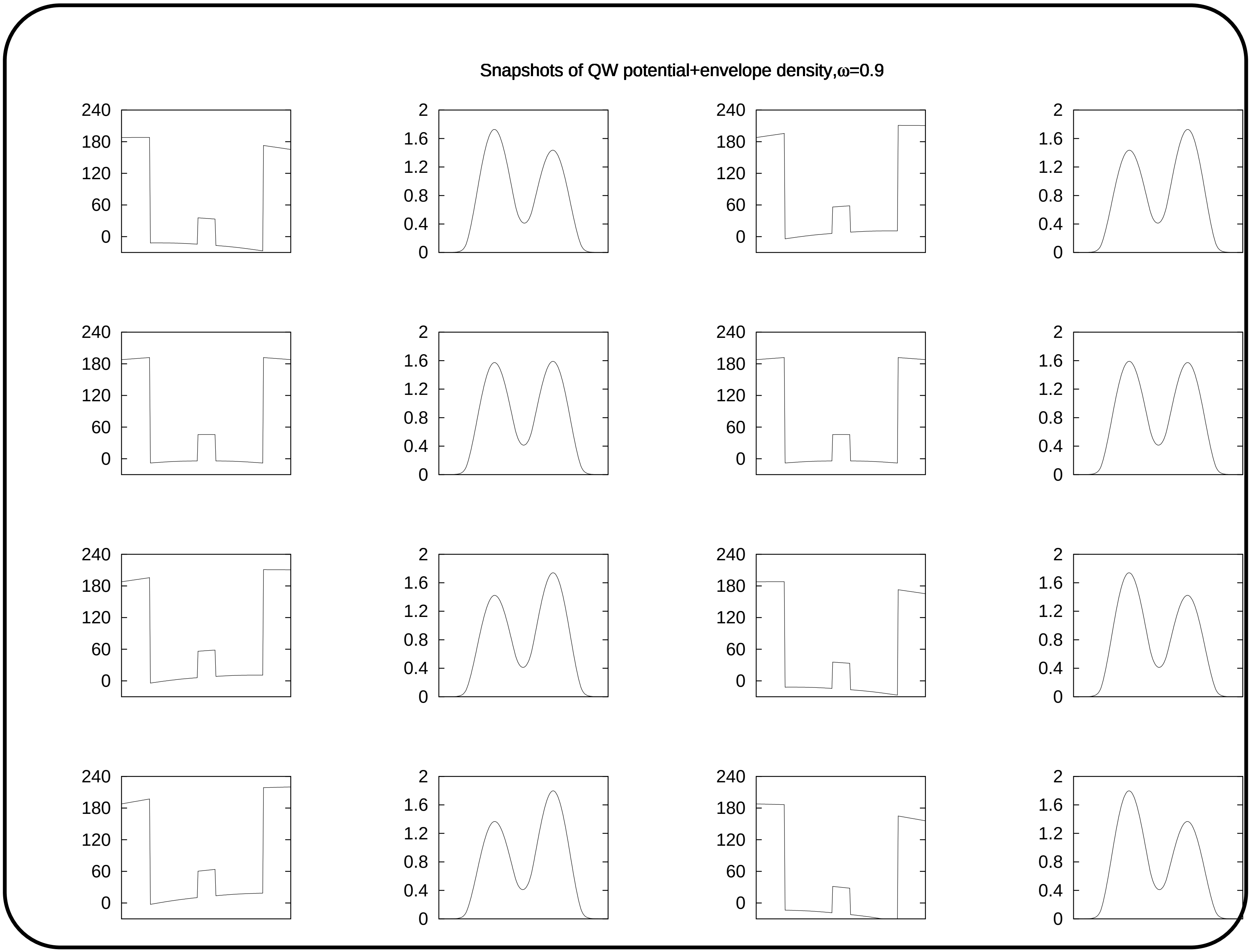


Absorption lines in the asymmetric QW

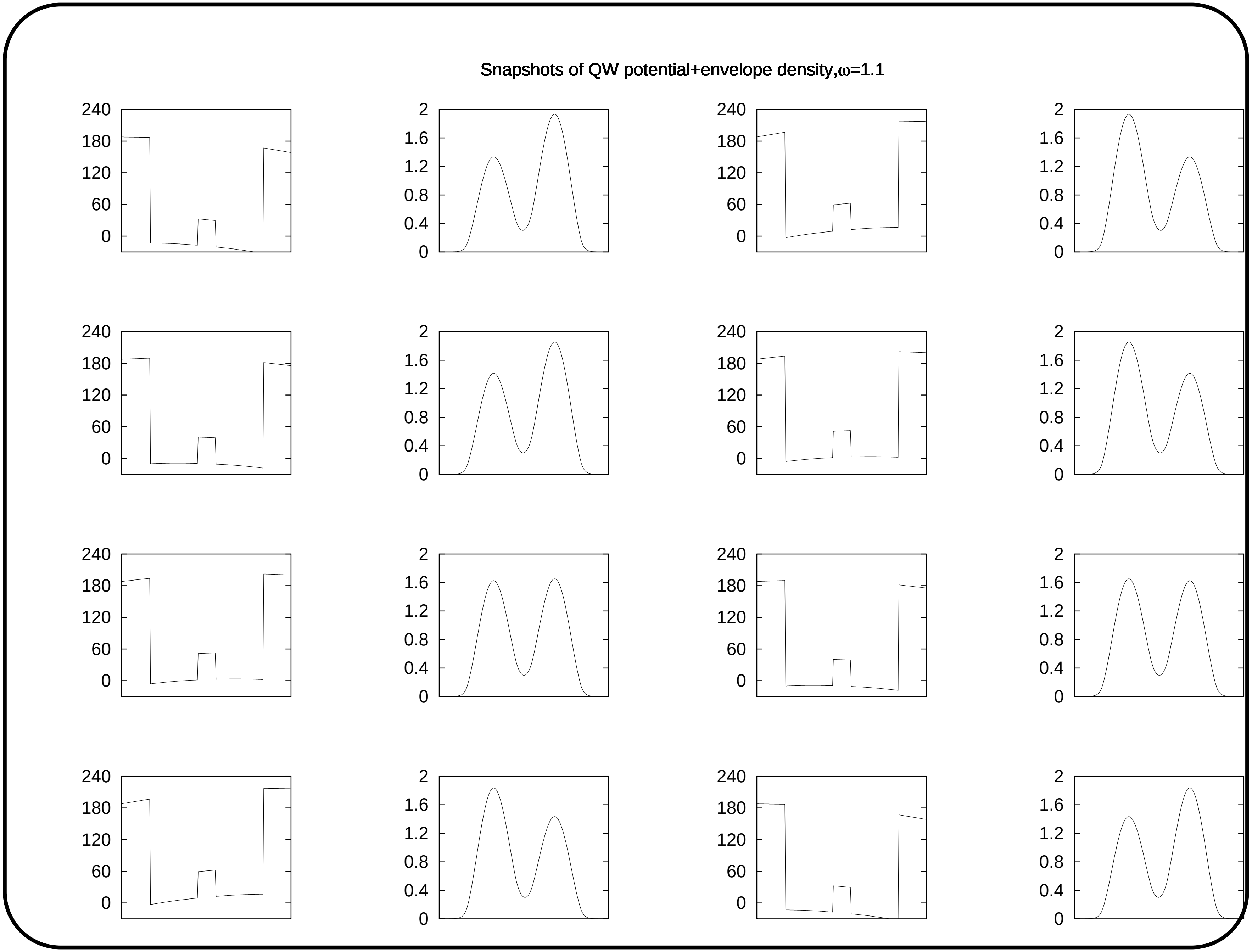




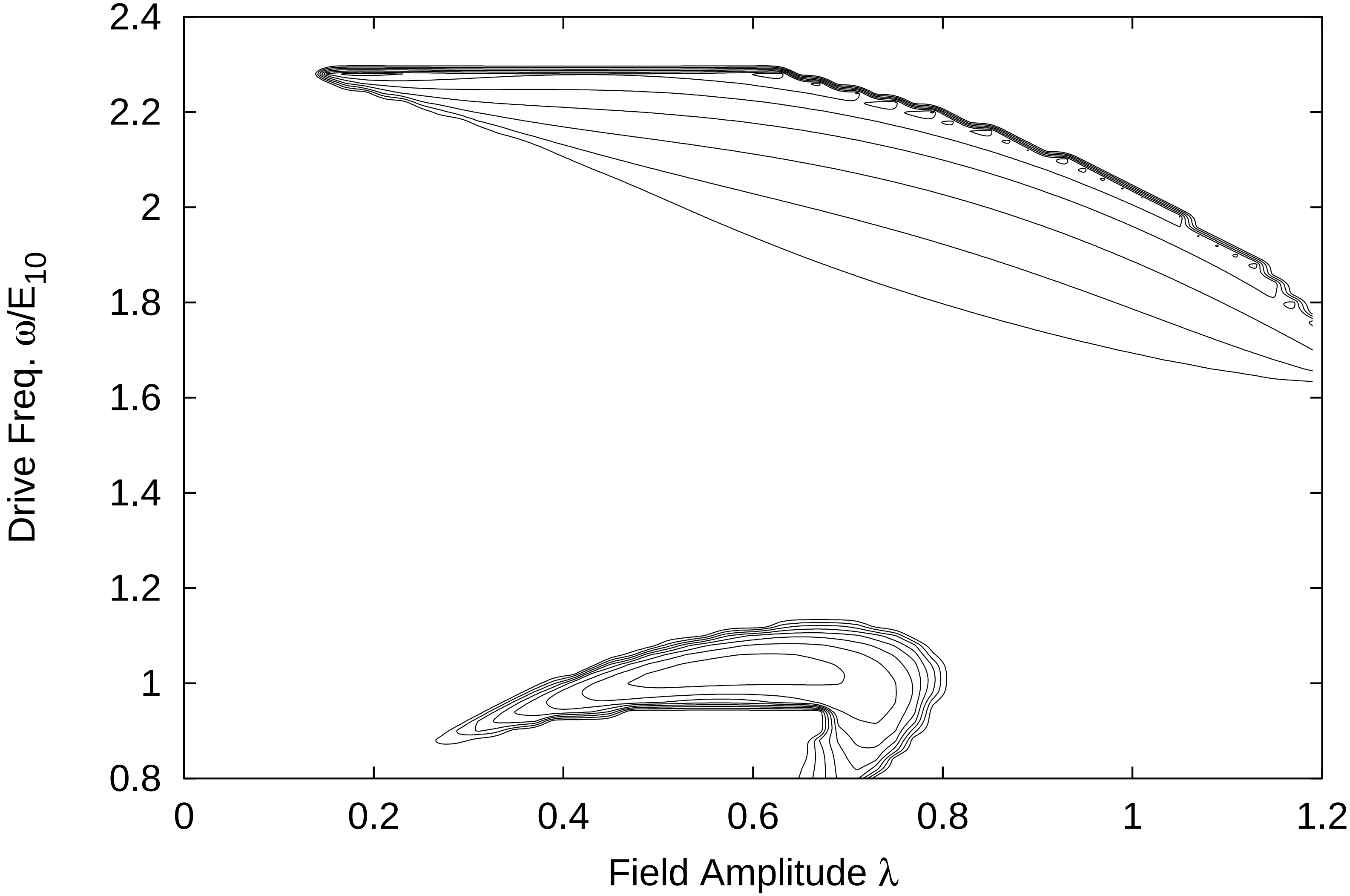
0.5
0.45
0.4
0.35
0.3
0.25
0.2
σ11
0
0.5
1
1.5
2
2.5
Field Amplitude in kV/cm
symmetric double QW
Ns= 1.0x10^11cm^-2

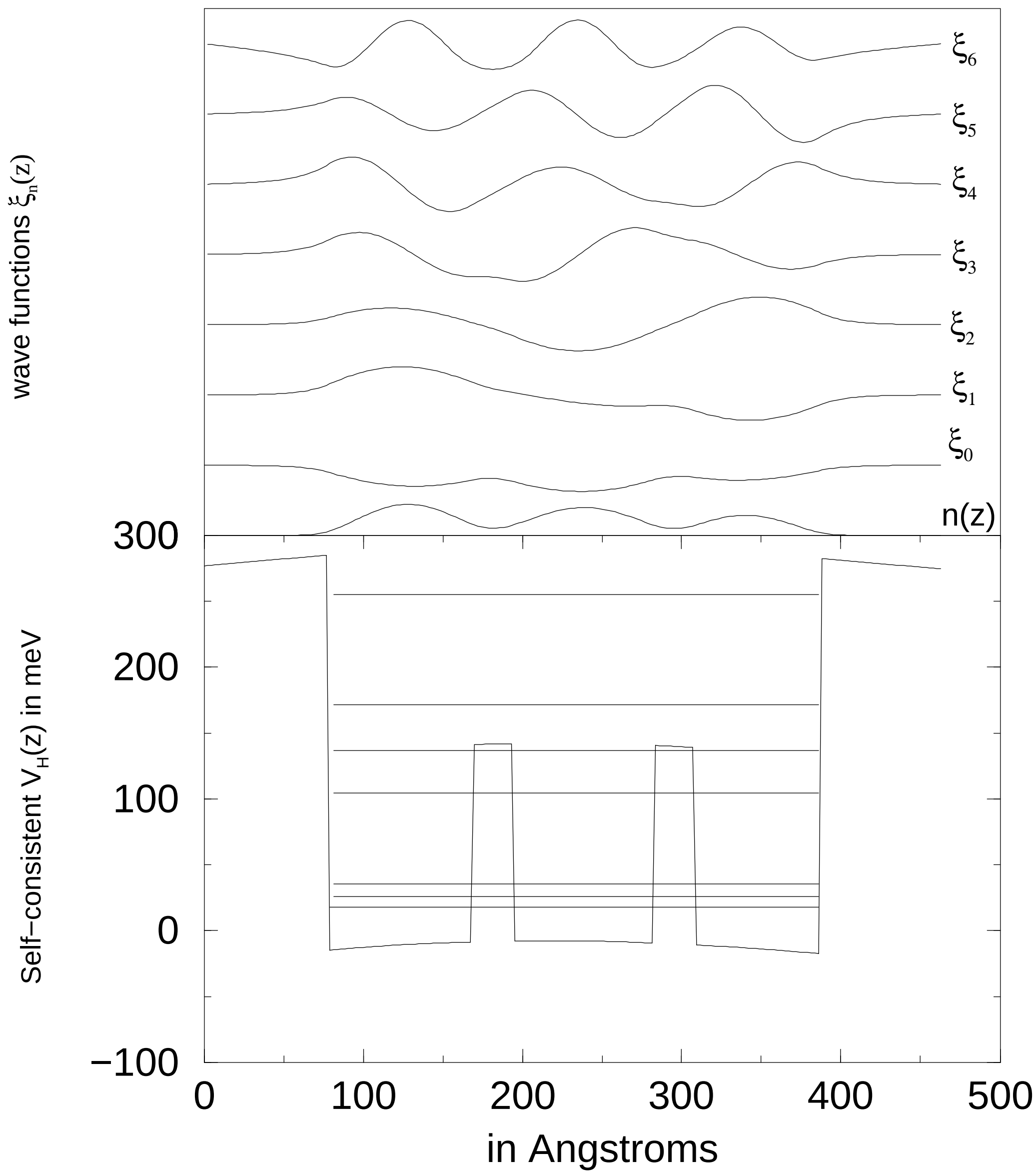


Snapshots of QW potential+envelope density,ω=0.9
240
180
120
60
0
2
1.6
1.2
0.8
0.4
0


Snapshots of QW potential+envelope density,ω=1.1
240
180
120
60
0
2
1.6
1.2
0.8
0.4
0

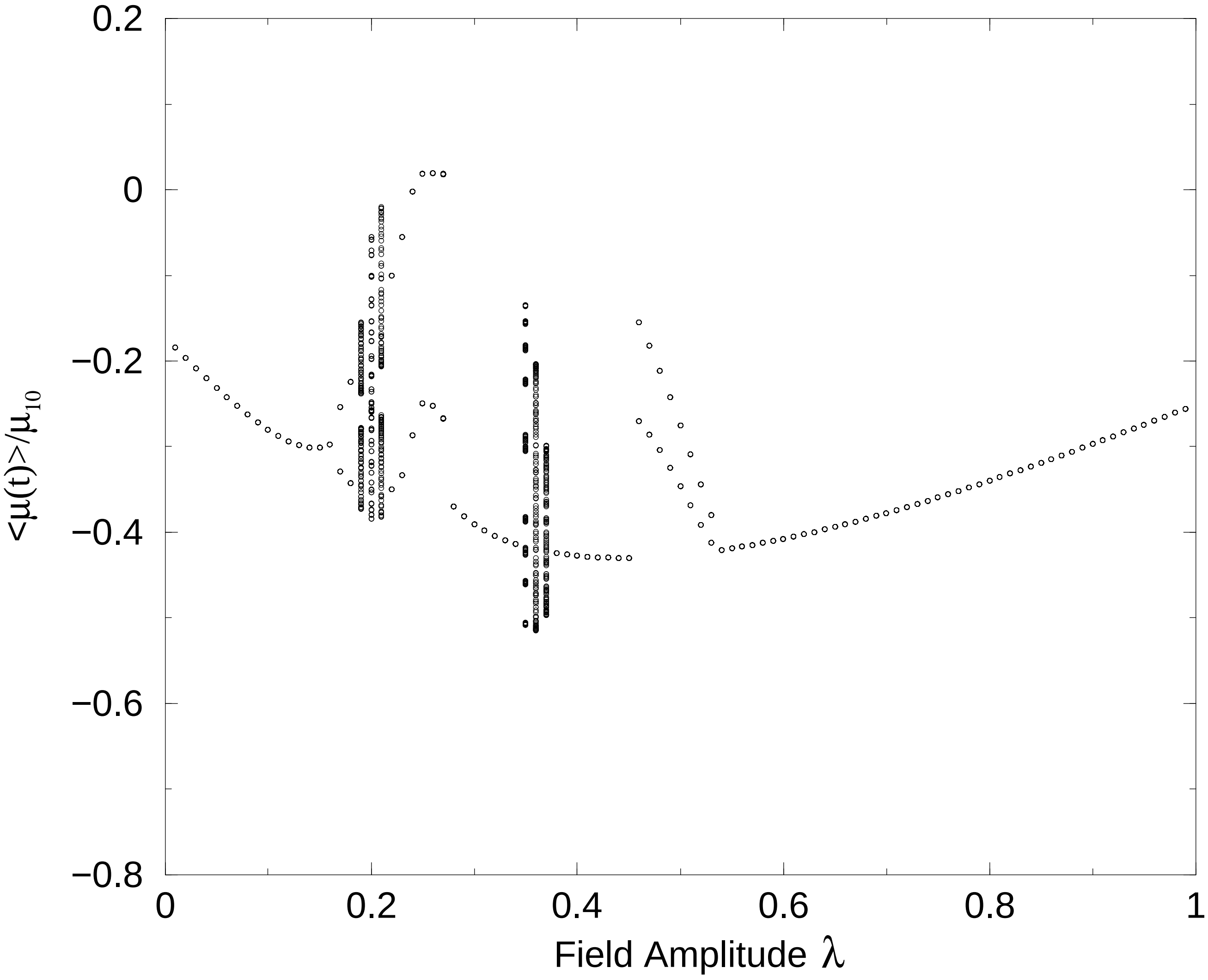
Period doubling bifurcation activity



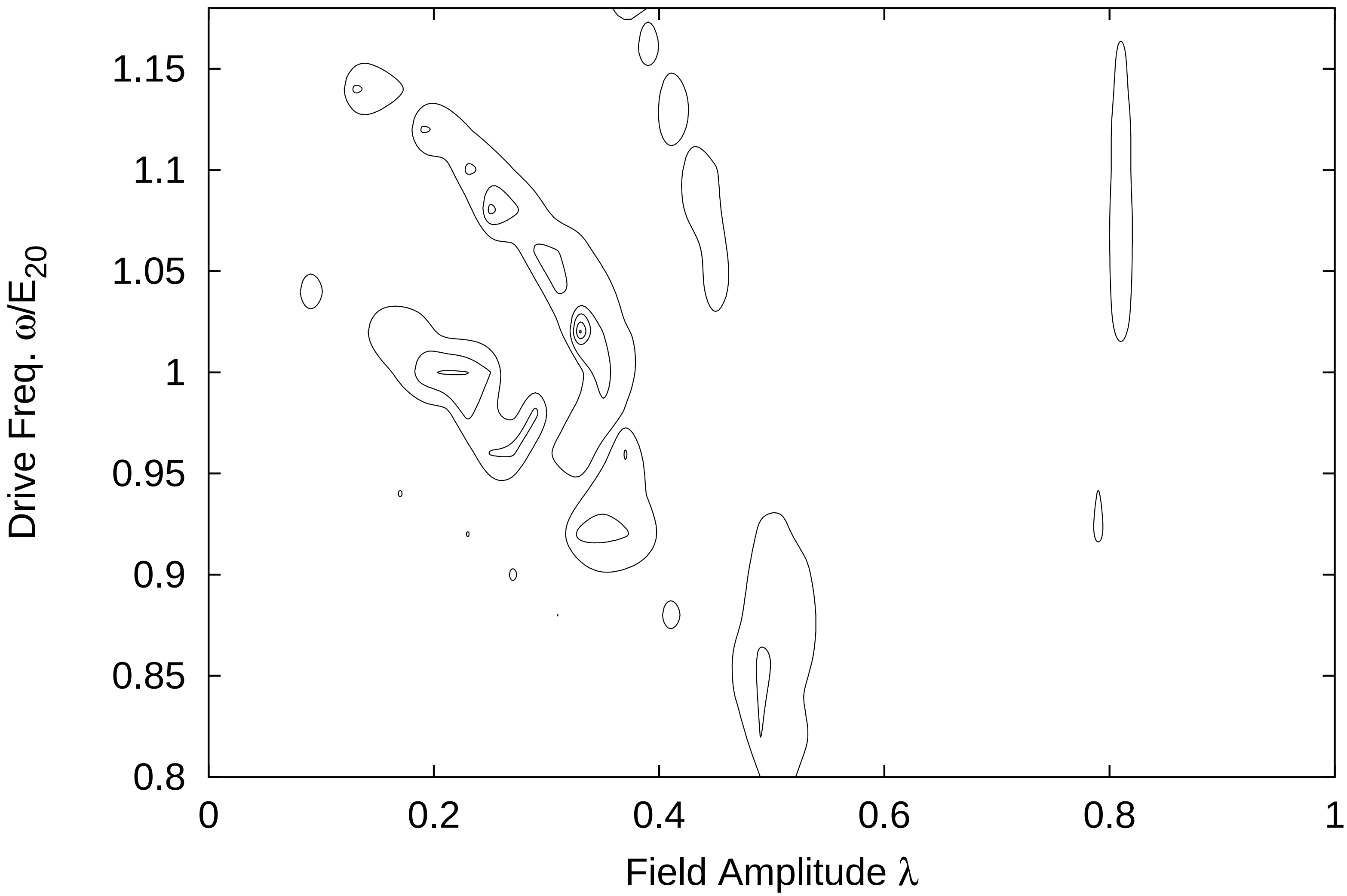


wave functions ξn(z)
ξ6
ξ5
ξ4
ξ3
ξ2
ξ1
ξ0
n(z)
Self−consistent VH(z) in meV
300
200
100
0
−100
0
100
200
300
400
500
in Angstroms

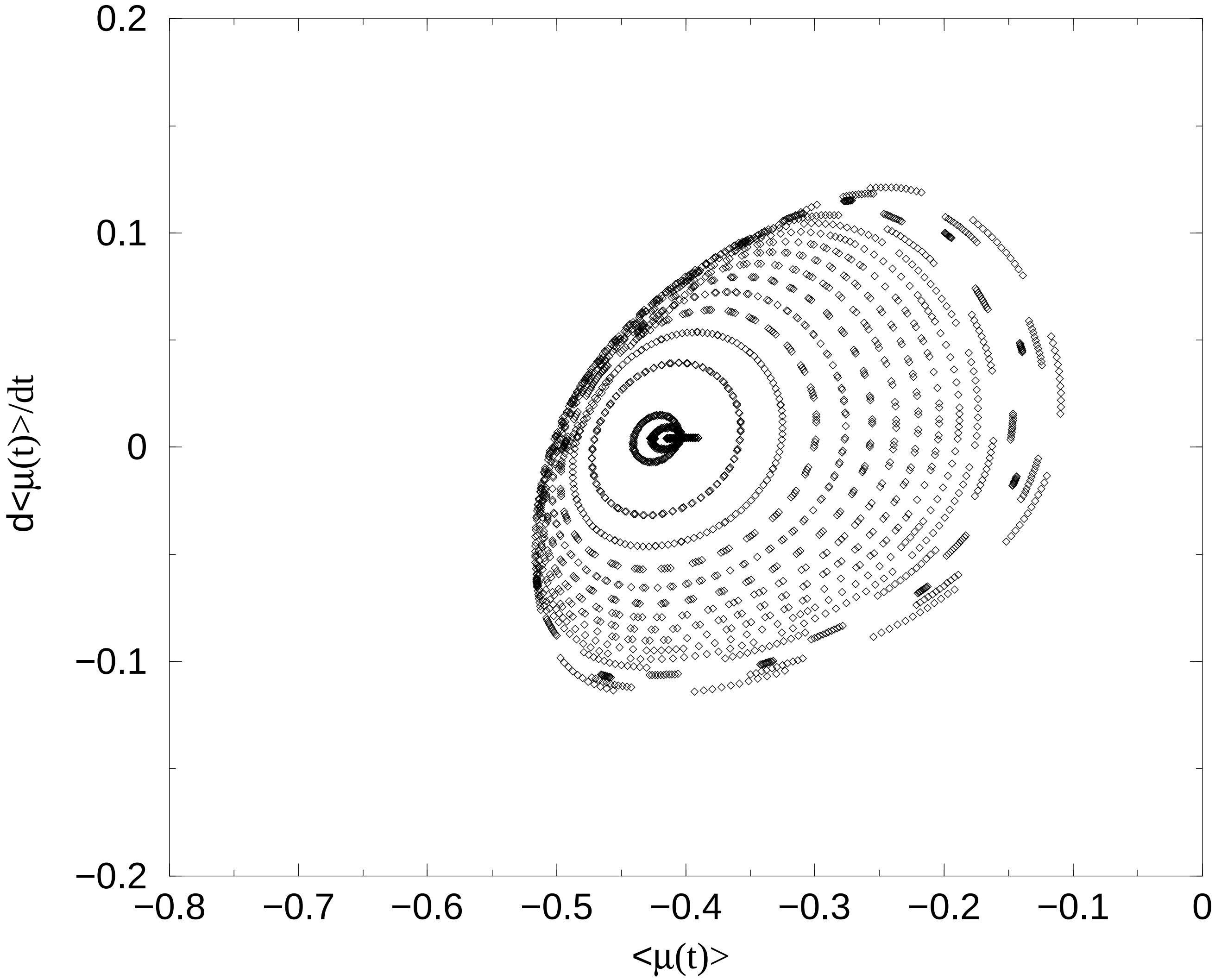
Diagram of Poincaré map for triple QW



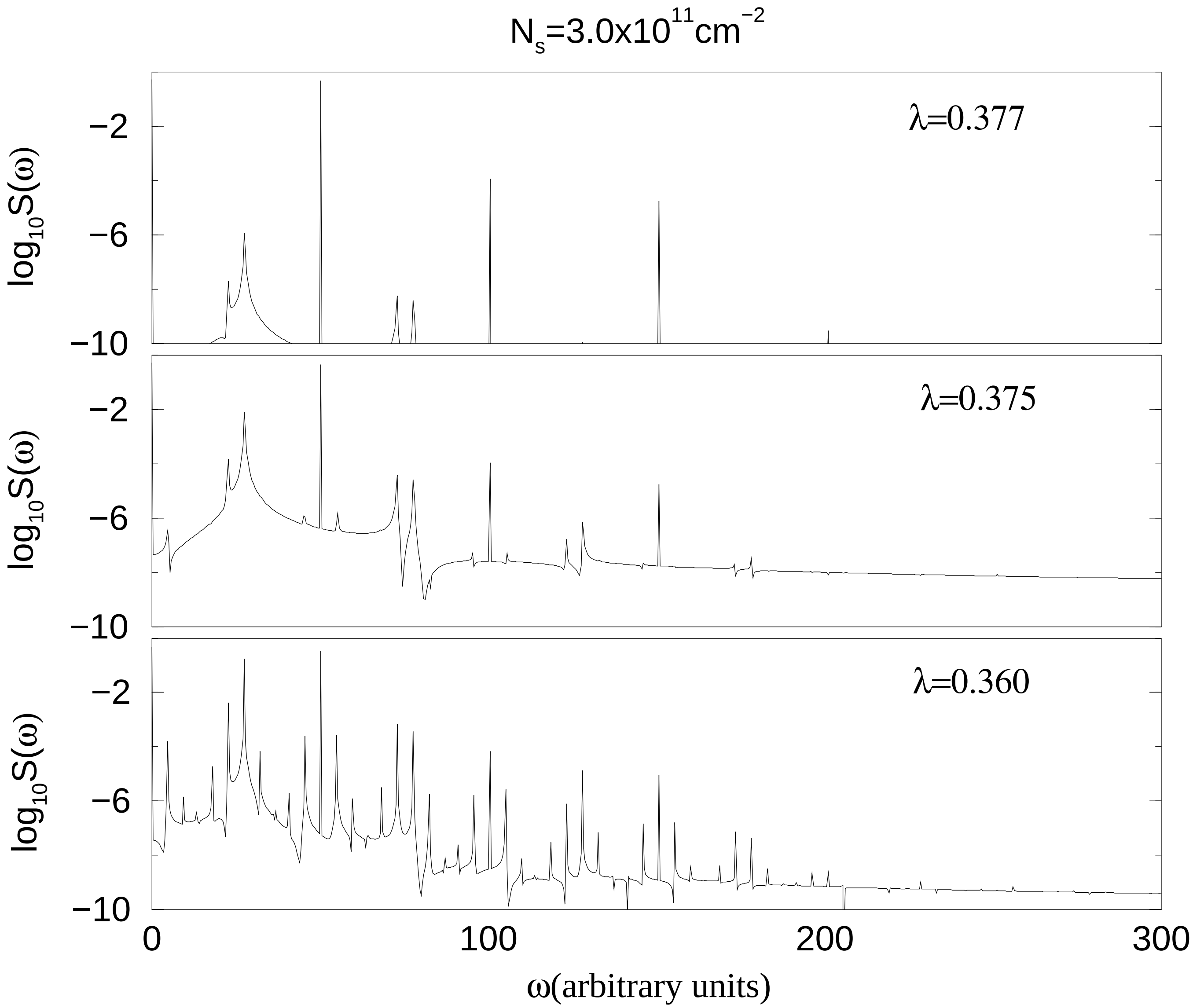
Range of nonlinear behavior for triple QW



Poincaré maps of Hopf bifurcation



Spectrum of response accross the Hopf bifurcation



Conclusions

- n-subband density-matrix equations of motion
- Averaging Method for the density-matrix equations
- Optical bistability at $\omega \approx \epsilon_{10}$
- PDB at $\omega \approx 2\epsilon_{10}$
- Hopf bifurcation for the three-subband n-doped QW