Universidad Carlos III de Madrid

# Seminario del Instituto Gregorio Millán

## *The RBF-FD Method: Developments and Applications*

**Víctor Bayona Revilla**
Universidad Carlos III de Madrid

Abstract

Radial Basis Function (RBF) methods have become a truly meshless alternative for the interpolation of multidimensional scattered data and the solution of PDEs on irregular domains. Its dependence on the distance between centers makes RBF methods conceptually simple and easy to implement for any dimension or shape of the domain. There are two different formulations for the solution of PDEs: the global RBF method and the local RBF method.

The global RBF formulation yields dense differentiation matrices which are spectrally convergent independently of the node distribution. Its principal drawback is that, as the overall number of centers increases, the condition number of the collocation matrices increases in a way that considerably restricts the applicability of the method. To overcome some of the drawbacks of the global RBF method, the local RBF method was independently proposed by several authors (also known as RBF-FD). Unlike the global RBF method, the RBF-FD method lacks spectral accuracy. However, the main feature of the method is the ability for handling irregular domains using highly sparse differentiation matrices while approximating the differential operators to high order. In this thesis we focus on the RBF-FD method.

In the first part we analyze the convergence properties of the method, obtaining novel analytical formulas for the local truncation error as a function of the shape parameter, inter-nodal distance and stencil size. This result proves the existence of a range of values of the shape parameter for which RBF-FD are more accurate than FD methods. Indeed, it usually exists an optimal shape parameter for which the local truncation error cancels out and the approximation becomes exact. To leading order, such a value is independent of the inter-nodal distance and only relies on the function and its derivatives. These results allow us to develop novel algorithms for the selection of the shape parameter in the solution of PDEs. In this line, two different strategies are proposed: a node-independent shape parameter, which minimizes the norm of the global error, and a node-dependent shape parameter, which minimizes the local truncation error at each node of the domain. Both strategies have been applied to the solution of classical elastostatic problems and it is shown that their accuracy with respect to FD methods can be significantly increased in one or two orders of magnitude by efficiently tuning the values of shape parameters.

The applicability of the method is explored in the second part of this thesis. This way, a three-dimensional problem for the propagation of a premixed laminar flame through a duct, has been solved. The good performance of the method inspires us to implement an RBF-FD method for the numerical study of an idealized Wankel microcombustor, whose geometry is more complex. The combustible flow field is modeled through the steady Navier-Stokes equations and the combustion process follows the combustion model above.

Día y hora: Lunes, 24 de junio de 2013 a las 12:30 horas
Lugar: Sala 2.1.C19 (Edificio Sabatini), Universidad Carlos III