The Statistical Theory of Turbulence

In this course we will disuss the turbulence problem and its significance. We start with the Navier-Stokes equation for fluid flow and discuss the role of fluid instabilities and noise. Then we give some background in probability theory that suffices to develop the form of the noise in fully-developed turbulence. Adding this noise to the Navier-Stokes equations results in the equation describing fully-developed turbulence: the stochastic Navier-Stokes equation. Then we develop the Kolmogorov-Obukhov statistical theory of turbulence. First by introducing more tools from probability theory, the Feynmann-Kac formula, Girsanov's theorem and Lévy processes and then by developing the She-Leveque intermittency correction to the scaling exponents of the structure functions of turbulence. The intermittency corrections turn out to be given by Log-Poisson processes. We show how to use these tools to compute the structure functions and determine their scaling. All the statistical quantities of turbulent fluids that can be measured or simulated are determined by the invariant measure of the stochastic Navier-Stokes equation. We show how to compute the Kolmogorov-Hopf equation determining this measure and solve it for the invariant measure of turbulence. Then we project the invariant measure onto the probability density function (PDF) of the velocity differences of turbulent fluids. The PDF is then compared with PDFs determined by experiments and simulations. If time permits we will discuss the extension of theory to boundary value problems.

The textbook will be: *The Kolmogorov-Obukhov Theory of Turbulence* by B. Birnir, Springer Verlag, New York, 2013, ISBN: 978-1-4614-6261-3 (Print) 978-1-4614-6262-0 (Online). http://link.springer.com/book/10.1007/978-1-4614- 6262-0