Microwave Imaging for Early Breast Cancer Detection
Using a Shape-based Strategy

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Abstract

In this paper we analyze the potential of a shape-based model for the early detection of breast tumors from microwave data. The tumors are modeled using a level-set technique. The formulation as a shape-reconstruction problem offers several advantages compared to more traditional pixel-based schemes, to mention in particular well-defined boundaries and the incorporation of an intrinsic regularization that reduces the dimensionality of the inverse problem whereby at the same time stabilizing the reconstruction. We present in this paper a novel strategy that is able to detect very small tumors compared to the wavelength used for illuminating the breast. In addition, our algorithm is able to determine the sizes and the dielectric properties of the tumors with good accuracy. Numerical experiments are shown in 2D which demonstrate the performance of this new technique in realistic situations.

1 Introduction

Microwave tomographic imaging is showing significant promise as a new technique for the early detection of breast cancer. Its physical basis is the high contrast between the dielectric properties of the healthy breast tissue and the malignant tumors at microwave frequencies [1, 2]. Making use of this characteristic, microwave imaging systems aim at detecting, localizing and characterizing tumors in the breast (see, for example, [3] and references therein).

The mathematical reconstruction problem in microwave tomography typically is treated as a nonlinear inverse problem in which a given cost functional is minimized via an iterative algorithm. Traditional iterative algorithms, well suited for nonlinear inverse problems and based on pixel reconstruction techniques, turn out to suffer from several drawbacks in this application. We mention in particular the typical oversmoothing effect of the interfaces between the tumors and the surrounding tissue in the reconstruction which is due to the need of strong regularization, normally addressed by adding a Tikhonov-Philips term to the cost functional.

Recently, a new family of iterative methods has been developed for the reconstruction of images in many different applications such as diffuse optical tomography, electrical impedance tomography or reservoir characterization. These approaches formulate the problems at hand as shape reconstruction problems and are based on a level set representation of these shapes (see [4] and references therein). Following this approach, we will assume here during the reconstruction that the dielectric properties in the breast are piecewise constant with only few possible values, namely one for the skin, one for the healthy tissue (both of them corresponding to positive values of the level set function) and another for the tumor (corresponding to negative values of the level set function). Traditional level set methods usually start from an initial guess of the location and shape of the tumor, and an artificial shape evolution is carried out in order to reduce the mismatch between measured and computed data. Recently, we have introduced a new technique
which does not start with a given initial shape, but creates a shape during the early iterations automatically at a 'good' location indicated by the sensitivity structure of the data [5, 6]. This speeds up the reconstruction and helps avoiding local minima. In the recent work [5, 6], we have assumed that the permittivity values inside the tumors are known a priori such that only the location and shape of the tumors need to be determined from the data.

In the current paper we present an important generalization of this new method. In addition to the goal of (i) locating a tumor and (ii) estimating its size, we now also try to (iii) specify simultaneously its correct permittivity value, which can then be used as an indicator for the malignancy of the tumor.

2 Mathematical Model

2.1 The forward problem

In our numerical work, the breast is modeled as a heterogeneous 2D medium $\Omega$, where a layer of skin is surrounding the inhomogeneous breast tissue which might include a hidden tumor, see Fig. 1. In this case, illumination by TM waves created by a source term $q$ can be modeled by the scalar Helmholtz equation

$$\Delta u + \kappa(x)u = q(x) \quad \text{in } \Omega$$  \hspace{1cm} (1)

which describes the non zero component of the electric field $u$. The complex wavenumber $\kappa(x) = \omega^2\mu_0\epsilon_0 \left[ \epsilon(x) + i\sigma(x) \right]$ combines the relative dielectric constant $\epsilon$ and the conductivity $\sigma$.

In the shape-based approach we assume that $\kappa(x)$ is a piecewise constant function that can be written as

$$\kappa(x) = \begin{cases} \kappa_i & \text{inside } S \quad \text{where} \quad \psi(x) \leq 0 \\ \kappa_e & \text{outside } S \quad \text{where} \quad \psi(x) > 0 \end{cases}$$  \hspace{1cm} (2)

for a sufficiently smooth level set function $\psi$ modeling the shape of the tumor $S$. The boundary of the tumor, $\partial S$, consists of all the points where $\psi(x) = 0$ and the tumor itself consists of all points where $\psi(x) \leq 0$. The unknowns here will be the level set function $\psi$ defining the geometry of the tumor, as well as the value of the relative dielectric constant $\epsilon_i$ inside $S$. For simplicity, the other electrical properties, $\kappa_e$ and $\sigma_i$, are assumed to be known. Extensions to the case in which these constants are also unknown follow the same lines of reasoning and will be addressed in our future research.

2.2 Estimating location and shape of the tumor

To solve the shape reconstruction problem for the tumor we will follow a time evolution approach. The goal will be to find the right hand side $f(x, t, \psi)$ of an evolution law

$$\frac{d\psi}{dt} = f(x, t)$$  \hspace{1cm} (3)

for the unknown level set function $\psi$ such that during the evolution the cost functional

$$\mathcal{J}(\psi) = \frac{1}{2} \| \mathcal{R}(\kappa(\psi)) \|^2$$  \hspace{1cm} (4)
is reduced and, upon convergence, minimized. In (4), \( R(\kappa(\psi)) \) denotes the mismatch between the true boundary data and those calculated by the forward model using the parameter distribution \( \kappa(\psi) \).

Formally differentiating the least squares cost functional \( J(\kappa(\psi(t))) \) with respect to the artificial time \( t \) and applying the chain rule yields

\[
\frac{dJ}{dt} = \text{Re} \int_{\Omega} R'(\kappa)^* R(\kappa) (\kappa_e - \kappa_i) \delta(\psi) f(x, t) \, dx,
\]

where \( \text{Re} \) indicates the real part of the corresponding quantity. In (5), \( R'(\kappa)^* \) denotes the formal adjoint of the linearized residual operator \( R'(\kappa) \) and the expression \( R'(\kappa)^* R(\kappa) \) coincides with the pixel-based Frechét derivative of this problem. It can be computed efficiently by using an adjoint scheme. From Eq. (5) we can select a descent direction for the cost functional by choosing

\[
f(x, t) = -\text{Re} \left( (\kappa_e - \kappa_i) R'(\kappa)^* R(\kappa) \right) \quad \text{for all } x \in \Omega.
\]

We note that our search direction \( f(x, t) \) has the property that it can be applied even if there is no initial shape available when starting the algorithm. Therefore, it allows for the creation of objects at any point in the domain, by lowering a positive level set function until its values arrive at zero. This property is useful for avoiding certain types of local minima which often occur in level set formulations if these are solely based on the propagation of an already existing shape. See the discussion led in [4].

Numerically discretizing (3) by a straightforward finite difference time-discretization with time-step \( \tau > 0 \) and interpreting \( \psi^{(n+1)} = \psi(t + \tau) \) and \( \psi^{(n)} = \psi(t) \) yields the iteration rule

\[
\psi^{(n+1)} = \psi^{(n)} + \tau f(x, t), \quad \psi^{(0)} = \psi_0.
\]

In our algorithm we will use \( f(x) \) as in (6) in order to locate the tumor position. Once the tumor position is identified, we will introduce a narrowband around \( \partial S \) (mathematically representing an approximation to the Dirac delta function \( \delta(\psi) \) in (5)) which is more typical for level set implementations and which works well when a sufficiently good approximation for the shape has already been found. This procedure allows us to refine our reconstruction of the shape of the tumor.

### 2.3 Estimating electrical properties of the tumor

In the previous subsection we have assumed that the values corresponding to the electrical properties of the tumor and the surrounding tissue were given, although they did not necessarily match the real ones. In our two-step algorithm, the first step will apply the above described strategy with a low permittivity value (for example \( \epsilon_i = 15 \)). With this strategy, our algorithm finds the location and shape of a tumor which correspond to the used (but typically incorrect) electrical properties in the sense that they minimize the least squares cost functional (4) for these values. It is now natural to assume that, if there is any hope to reconstruct shape and parameters of the tumors simultaneously, this minimal cost value upon convergence will be the smaller the closer our probing value for the permittivity has been to the correct one. This assumption forms the basis of the second part of our two-step algorithm.
In the second part of our algorithm, we aim at simultaneously reconstructing shape and permittivity value of the tumor. After applying during the first step of the algorithm the procedure described above assuming a low permittivity value (for example $\epsilon_i = 15$), we now continue in an efficient manner with probing the corresponding minimal cost values which we would get for successively higher permittivity values. For this purpose, we increase the value of $\epsilon_i$ by a fixed small step-size $\delta \epsilon_i$, and repeat the above search for the shape starting from the reconstructed shape corresponding to $\epsilon_i = 15$ but now assuming the value $\epsilon_i = 15 + \delta \epsilon_i$. For this new search only very few (typically not more than 5) iterations are needed, since only a small correction of the current shape needs to be found. We then iterate this step continuing with the permittivity value $\epsilon_i = 15 + 2\delta \epsilon_i$ and using as starting guess the reconstructed shape for $\epsilon_i = 15 + \delta \epsilon_i$. We plot the achieved minima of the cost functional for each permittivity value in a graph against the corresponding permittivity value as shown in Fig. 1. Then, we search for the global minimum of this graph, and interpret the corresponding permittivity value as the recovered true value. Moreover, the shape corresponding to that permittivity value is interpreted as the recovered true shape.

3 Numerical Experiments

In order to test the numerical algorithm, we have implemented several test cases in a 2D tomographic configuration. We describe in the following two representative cases. In both we use a set of 40 transducers which are equidistantly located around the 12-cm-diameter breast. They illuminate the breast with five different frequencies: 500, 800, 1000, 1500 and 2000 MHz. The Helmholtz equation is solved with a second order finite differences scheme and a perfectly matching layer (PML) at the boarders of the computational domain. The grid consists of $160 \times 160$ pixels, each pixel having the physical size $1 \times 1$ mm$^2$.

We perform a loop over the set of frequencies 40 times in the first part of our algorithm, and 5 times over the frequencies for each value of the permittivity assigned in the second part. In all cases we fix the conductivity of the tumor to the predefined value. The surrounding medium is modelled as having the values $\varepsilon_{\text{liquid}} = 2.5$ and $\sigma_{\text{liquid}} = 0.04$ Siemens/m (S/m), the breast tissue has the parameters $\varepsilon_e = 9.0$, $\sigma_e = 0.4$ S/m, and the skin layer has $\varepsilon_s = 34.0$, $\sigma_s = 1.0$ S/m. The correct tumor has the values $\varepsilon_{\text{in}} = 36.0$ ($\varepsilon_{\text{in}} = 49.0$ in the second numerical experiment), $\sigma_{\text{in}} = 4.0$ S/m. To simulate the heterogeneity of human breast tissue, we add random variations to the homogeneous background permittivity values of up to ±5 per cent in magnitude. These take the form of $4 \times 4$ mm$^2$ squares which are distributed over the domain in a random fashion. In addition, the data in the detectors are perturbed by 1 per cent white Gaussian noise.

In Fig. 1 we have combined two figures showing the corresponding reconstructions. We show in the left figure a reconstruction for the situation where the tumor has the electromagnetic parameters $\varepsilon_{\text{in}} = 49.0$, $\sigma_{\text{in}} = 4.0$ S/m, and is embedded at a depth of about 2 cm beneath the skin. The size of the tumor is $63$ mm$^2$. In the right figure we show a reconstruction for the situation where the tumor has the electromagnetic parameters $\varepsilon_{\text{in}} = 36.0$, $\sigma_{\text{in}} = 4.0$ S/m, and is embedded slightly deeper than before, namely at a depth of about 2.4 cm beneath the skin. The size of the tumor is now only $47$ mm$^2$.

In both figures the left columns show (from top to bottom) the correct tumor, the reconstruction at the end of the first part of our algorithm (achieved by assuming the typically incorrect
starting permittivity value of $\varepsilon_{in} = 15.0$), and the reconstruction in the second part of the algorithm which corresponds to the permittivity value with the lowest cost value. In the right columns of the figures we show (from top to bottom) the evolution of the cost functional with respect to the permittivity value (recall that we iterate over the permittivity value in this second part), the evolution of the difference in size (number of pixels) between the true tumor and the reconstructed one at the given iteration (permittivity value), and finally the size of the symmetric difference between the true and the reconstructed tumor.

We observe that the cost functional shows in all of these cases its absolute minimum at a value of the permittivity which is very close to the correct one. Therefore, the reconstructions shown in the bottom left image of the figures provide us with a very good estimate not only of the location and the shapes of the tumors, but also of the correct permittivity values. In order to verify the agreement in location and size of the reconstructed tumors at each value of the probing permittivity value, we have displayed the two additional graphs in the right columns of the figures showing difference in size and absolute value of the symmetric difference. Also these two graphs
demonstrate a very good agreement between our reconstruction and the true tumor.

4 Conclusion

We have presented a shape-based algorithm for the early detection of breast tumors from microwave data. The algorithm consists of two parts. The first part does not employ a narrow-band function in the level set representation of the tumor and is mainly aiming at localizing the tumor and finding a good first guess of its size. We experimentally find that this location of the tumor is quite robust and does not depend very much on the value of the relative dielectric constant $\epsilon_i$ assumed for the probing tumor (we use here $\epsilon_i = 15$) as long as it is sufficiently higher than the background value $\epsilon_e$. The second part of the algorithm starts from the reconstruction achieved in the first part and continues with a refined simultaneous search for contrast and shape of the already located tumor. For this part we apply a narrow-band strategy, since no new tumors far away from the already found one are of interest in this part. In our numerical experiments (performed here in a 2D setup) we demonstrate that this two-step algorithm is able to reconstruct good approximations to the location, shape and correct permittivity value of a small hidden tumor from noisy microwave data even though the algorithm does not take into account the background fluctuations of the physical parameters in the breast.

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References


