Chaotic behavior in a one-dimensional cardiac model

David G. Schaeffer[1]

[1]*Department of Mathematics, Duke University, Durham, NC 27708, USA*

In electrocardiology, the term action potential refers to the behavior that, in response to a brief stimulus, the electrical potential across cardiac cell walls is elevated for an extended period. The duration of action potentials under periodic pacing is an important quantity clinically, physiologically, and mathematically. At slow to moderate pacing rates, every stimulus produces an action potential of the same duration, but at high pacing rates cardiac tissue often undergoes a bifurcation to what is called alternans: i.e., uniform APDs are replaced by an alternation between short and long action potentials. In a single cell or a small piece of cardiac tissue, this bifurcation is a familiar period-doubling bifurcation, but when propagation effects are important the nature of the bifurcation to alternans is far from clear. For example, the short/long alternation may suffer phase reversals at various locations in the tissue. This behavior, known as discordant alternans, is considered to be a precursor to ventricular fibrillation. In collaboration with my student, Shu Dai, I have studied these phenomena through an approximate equation (derived by Echebarria-Karma) for the modulation of nonuniform wave trains in one spatial dimension. In this lecture, after describing the context of the problem, I will report on this work. In particular, we have shown that: The modulation equation undergoes both Hopf and steady-state bifurcations; which bifurcation occurs first depends on a parameter derived from the speed of traveling waves; the competition between the two modes gives rise to interesting secondary bifurcation. Moreover, in certain parameter ranges, solutions of this equation exhibit chaotic behavior, as indicated in my title. While chaos has been observed in simulations of spiral waves in several dimensions, it is surprising to find it in a one-dimensional model.