Ensemble-Averaged and Deterministic Descriptions of Tumor-Driven Angiogenesis

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A stochastic model for the growth of blood vessels (angiogenesis) attracted by a tumor is discussed. According to this, tips of newly created capillaries feel a chemotactic force proportional to the gradient of a growth factor released by the tumor, thus they move, possibly branch out, and may melt with other vessels (anastomosis). Even though the latter process keeps the number of tips relatively low, a deterministic description of the vessel tip density can be found by means of ensemble averages over many replicas of the angiogenic process. A good agreement between the two models is achieved as far as the anastomosis rate is properly fitted. On the other hand, the average advance of vessel tips towards the tumor can be described by a soliton, whose features can be controlled in terms of few parameters.

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