Seminario del Instituto Gregorio Millán

Well-posed and ill-posed regimes in μ(I)-rheology for granular materials

David Schaeffer
Duke University
(collaboration with P. Bohórquez and N. Gray)

Abstract

Progress in understanding granular flow has been greatly hampered by the lack of satisfactory constitutive equations. Historically, the concept of a *Coulomb material*, based on rate-independent plasticity, was introduced to describe granular materials. On substitution into the equations for conservation of mass and momentum, this constitutive relation leads to a system of evolution equations loosely analogous to the Navier-Stokes equations; friction gives rise to a term that formally resembles viscosity. However, it turns out that *this system is ill-posed*. Numerous higher-order, non-local theories have been introduced in an attempt to resolve this difficulty; while many of these are well-posed, they are invariably quite complicated, perhaps unnecessarily so.

In the last decade the French school (GDR MIDI) proposed a natural modification of the Coulomb constitutive equation. In this theory the coefficient of friction varies with the shear rate (which is measured by a nondimensional inertial number *I*); this property leads to the name *μ(I)-rheology*. Their equation, which is based on experiments of flow down inclined planes and on dimensional analysis, retains a level of simplicity comparable to Coulomb material.

In this talk we analyze the well-posedness of the governing equations using *μ(I)-rheology*. Specifically, we show that these evolution equations are well-posed for a large range of deformation rates but become ill-posed at extremes of slow or fast deformation. It is known that additional effects, not represented in *μ(I)-rheology*, become important in these two extremes. Thus, the present mathematical result and physical understanding of granular materials support one another.

On the numerical side, several authors have adapted a recently proposed finite volume method for solving the Navier-Stokes equations to problems with *μ(I)-rheology*. In this method, the pressure viscosity contribution is evaluated explicitly; this is appropriate for viscosity in the Navier-Stokes equations (where the viscosity operator is elliptic) but questionable for the not-necessarily-elliptic operator that occurs in *μ(I)-rheology*. Reflecting this mismatch, numerical results using this method show no indication of ill-posedness: i.e., they do not reproduce the stability properties of the PDE derived assuming *μ(I)-rheology*. To better capture the behavior of the PDE, we propose a PISO-like method that evaluates implicitly the viscous pressure contributions, and we derive a new pressure equation based on the Schur complement. We present numerical simulations to illustrate that our method does capture ill-posedness as predicted by theory.

Día y hora: Viernes, 5 de abril de 2013 a las 12:30 horas
Lugar: Sala 2.1.C19 (Edificio Sabatini), Universidad Carlos III